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ABSTRACT

Sovereign credit ratings in the European Union: a model-based fiscal analysis

We propose a model-based measure of sovereign credit ratings derived solely from the fiscal position of a country: a forecast of its future debt liabilities, and its potential to use tax policy to repay these. We use this measure to calculate credit ratings for fourteen European countries over the period 1995-2012. This measure identifies a European sovereign debt crisis almost two years before the official ratings of the credit rating agencies.

JEL Classification: E62, H30 and H60
Keywords: credit risk, default probability, fiscal policy and sovereign risk

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1 Introduction

Motivation

In this paper we suggest a way of constructing a benchmark measure of sovereign credit ratings based solely on the fiscal position of a country that could be useful for investors, Credit Rating Agencies (CRAs) and in determining macroeconomic policy. The motivation for this is the close scrutiny that CRAs have come under since the onset of the financial crisis in 2008. Following the credit downgrades of a number of eurozone sovereigns in mid 2011 and early 2012, CRAs have been accused of exacerbating the eurozone debt crisis and of contributing to a rise in the cost of borrowing above sustainable levels for several European countries. As a result, since the financial crisis, macroeconomic policy in many European countries appears to have been concerned more with its sovereign credit rating that with moving the economy out of recession.

In November 2011 the European Commission issued a proposal for stricter rules on CRAs to make them more transparent and accountable, and to increase competition in the credit rating sector. The Commission's proposal stressed the role of conflict of interests, political interference and inefficiencies in existing CRAs methodologies. It also suggested the creation of an European-based CRA to counter the influence of U.S.-based CRAs (European Commission, 2011).[1]

New regulations on CRAs were subsequently approved on January 2013 by the European Parliament. These allow agencies to issue unsolicited sovereign debt ratings only on set dates; make CRAs more accountable for their actions; and ensure that information on the underlying facts and assumptions on each rating is made publicly available in order to facilitate a better understanding of credit ratings (European Commission, 2013). Both the 2011 proposal and the 2013 regulations stressed the importance to financial investors of determining their own independent evaluation of credit ratings.[2] Subsequently, however, the Commission abandoned the plan of establishing a new (European-based) CRA as it was thought too costly.

This paper shows that it is feasible to construct a model-based measure of sovereign credit ratings that is transparent, independent and timely. Transparency refers to the ease of general public to access and to reproduce credit ratings and to the ability of the public to make its own judgment about their validity. Independence reflects the derivation of sovereign credit ratings that are model-based rather than driven by the subjective evaluation of analysts. The rating can be updated systematically using the latest available data and, for this reason, is timely. The measure is inexpensive to produce, and can even be automated. We then use this procedure to provide credit ratings for fourteen European countries over the period 1995:4-2012:4.

[1] The role of asymetric information and conflict of interests in the credit-rating industry has been extensively analysed in the economic literature. Recent examples include Mathis, Mc Andrews and Rochet (2009) and Bolton, Freixas and Shapiro (2012).

[2] White (2010)'s review of the regulatory structure of CRAs concludes with a similar proposal of investors seeking their own independent assessment of the credit rating as a way for reducing reliance on CRAs.

Contribution

The proposed credit rating measure is computed by adapting to sovereign debt the logic of Black and Scholes's (1974) formula for pricing credit risk, as calculating a credit rating is similar to pricing the probability of exercising an American option. It entails estimating the probability that the debt-GDP ratio will exceed a given limit or threshold over a given time horizon and then mapping this default probability into a credit rating. Uncertainty about the credit rating can be taken into account using estimates of the distributions of the forecast error of the debt-GDP ratio and of the debt limit. This provides a general framework for constructing sovereign credit ratings that can be implemented using any forecast or official budget projections of the distribution of the debt-GDP ratio and any measure of the debt limit.

In this paper we form the forecast of the debt-GDP ratio using an open economy reduced-form model that allows for time variation in its parameters due possibly to structural or policy changes, and has time-varying volatility. This is based on rolling-window estimation of a VAR, i.e. a ROVAR model. The ROVAR is chosen because it can be easily estimated and updated. The debt limit we use measures the maximum borrowing capacity of an economy derived from a dynamic stochastic general equilibrium model (DSGE) of an open economy model that features an exogenous default probability on sovereign bonds and distortionary taxation. The debt limit is based exclusively on the ability of a government to alter fiscal policy in the future to meet its outstanding financial obligations. This depends on whether fiscal policy changes are anticipated or unanticipated by market participants and, if unanticipated, whether they could arise from changes in expenditure policy, tax policy or both. The model is solved using a nonlinear algorithm calibrated with time-varying and country-specific data. This delivers time series of the debt limit that show how the maximum borrowing capacity of an economy evolves over time as a result of the changing ability of a government to use its fiscal instruments to repay its financial obligations and of changes in the state of the economy.

Basing the debt limit solely on fiscal considerations provides a narrow assessment of sovereign creditworthiness that excludes other factors that might contribute to the ability of a government to repay debt, such as the willingness and the political ability of delivering the required changes in fiscal policy, or the possibility of using either domestic or external non-fiscal sources of debt repayment, for example, changes in monetary policy and external bailouts. The merit of this narrower but simpler definition is that it conveys a clear and unambiguous interpretation of the credit rating, a feature particularly relevant for investors seeking transparent and independent assessment of the credit ratings. Any discrepancies between the model-based and the official ratings could therefore be due to the CRAs taking into account factors beyond the mere financial ability of generating saving to repay debt. The methodology outlined in the paper can be extended to include some, if not all, of these non-fiscal factors but would be at the expense of further complicating the cross-country analysis and the interpretation of the determinants of the credit rating.

There is a substantial academic literature on the theoretical determinants of sovereign default risk and empirical research on how well financial and macroeconomic variables explain official sovereign credit ratings,[3] but it appears that there is little or no literature on how one might construct sovereign credit ratings based on macroeconomic fundamentals. The literature cannot therefore provide an assessment of sovereign credit ratings that is independent of the credit ratings of the CRAs.

Empirical findings

The fourteen European (EU14) countries for which we compute model-based measures of their sovereign credit ratings are Austria, Belgium, Denmark, Finland, France, Germany, Greece, Ireland, Italy, the Netherlands, Portugal, Spain, Sweden and the U.K.[4] The model-based credit ratings are then compared with the historic credit rating issued by CRAs and with market-determined sovereign credit default swap (CDS) prices.

The historic credit ratings for the EU14 countries over the past 20 years have been somewhat higher than those of other countries. Their cross-section distribution has been stable within the investment grade at least until 2010. At this point the distribution became more dispersed signalling the start of the EU sovereign debt crisis. There is, however, no clear relation between changes in the ratings issued by CRAs during the financial crisis and the market's perception of the probability of sovereign default as measured by changes in CDS prices. In fact, a number of countries have received the highest credit rating despite fluctuations in their CDS prices. In contrast, other countries have been downgraded either after a significant increase in their CDS prices, or even when their CDS prices were falling.

The main findings are that the model-based credit ratings: (i) anticipate the downgrades of Ireland, Spain, Portugal and the U.K. that occurred from the end of the 2010s; (ii) downgrade Greece to the lowest rating (coinciding with its highest default probability) from at least mid 2000; (iii) suggest that the Italian sovereign credit rating has been overstated. For all other countries, the model-based credit ratings are similar, but not identical, to the credit ratings provided by the CRAs as the model-based credit ratings indicate temporary downgrades of 1 or 2 notches for short periods of time (1 or 2 quarters) whenever there is a temporary deterioration in the fiscal stance. An implication of these results is that the cross-section distribution of the model-based sovereign credit rating is no longer concentrated within the investment grade prior 2010 and it starts changing significantly from 2008. This suggests that a model-based credit rating would have identified and signalled to market participants signs of the impending European sovereign debt crisis well before 2010, when the CRAs first reacted to the crisis. The historic ratings may differ from the model-based ratings, especially in the early stages of the crisis, not because their response is delayed, but because they take account of additional factors to those involved in

[3] Recent examples of this include Hill, Brooks and Faff (2010), Afonso, Gomes and Rhother (2011) and Afonso, Furceri and Gomes (2012).

[4] See Polito and Wickens (2012b) for an application to the U.S. using a closed-economy version of the structural model for the debt limit.

determining fiscal savings.[5] We also find that for several countries the model-based credit ratings anticipate the changes in CDS prices that occurred during the financial crisis.

A by-product of the methodology proposed in this paper is the quantification of a country's debt limit (measured as its maximum borrowing capacity) and how this changes over time. The numerical analysis suggests that for most EU14 countries the scope for increasing borrowing capacity by increasing taxation is limited as actual tax revenues are similar to tax revenues maximized with respect to tax rates. Our findings suggest that EU14 countries are more likely to be able to raise debt limits and achieve fiscal consolidation by reducing their expenditures than by increasing taxes.

Paper organization

In Section 2 we provide some information about the sovereign credit ratings issued by the CRAs and establish a number of stylized facts about the historic credit ratings of EU14 countries. In Section 3 we describe the theory underlying the model-based sovereign credit ratings. The DSGE macroeconomic model used to derive debt limits is developed in Section 4, and we report the numerical solutions for the EU14 countries for the period 1995:4 to 2012:4. In Section 5 we report the model-based sovereign credit ratings for the EU14 countries and re-evaluate the stylized facts outlined in Section 2. We reflect on our findings and discuss potential extensions of this approach in Section 6. The data used in the paper are described in Appendix A; the theoretical derivation of the debt limits is summarized in Appendix B; and the algorithm used to numerically evaluate country debt limits is described in Appendix C. Further results on the model-based ratings are in Appendix D.

2 Historic rating of EU14 countries

Sovereign credit ratings are opinions issued by CRAs on the creditworthiness of a particular sovereign issuer or financial instrument. They assess the likelihood that a sovereign government will default either on its financial obligations generally (issuer rating), or on a particular debt or fixed income security (instrument rating).

The notion of a sovereign credit rating has evolved over time. Originally it was based on the perceived ability and willingness of a government to meet its financial obligations. More recently the three main CRAs (Fitch Ratings, Moody's Investors Service and Standard & Poor's) view a sovereign credit rating as being closely related to a government's ability to repay debt. This definition seem particularly appropriate for countries that, like most of the EU14,

[5] Under this alternative interpretation, the most likely factors to explain the difference between the model-based and the historic ratings are (i) the ability of using domestic monetary policy (inflation) to complement fiscal revenues for countries that are not in the Euro and (ii) the confidence in the possibility of the European Central Bank (ECB) becoming a lender of last resort (or equivalently confidence in the willingness of maintaining the common currency) for countries in the Euro.

are generally regarded as being committed to the repayment of their sovereign obligations.

The methodologies used by CRAs to determine sovereign ratings are ultimately based on the judgment of their teams of analysts. No CRA uses a mathematical formula or an economic model to measure sovereign credit ratings. Instead, sovereign risk units are in charge of issuing new credit ratings and of monitoring and reviewing existing ratings. The qualitative and quantitative criteria and variables employed to determine a credit rating vary across CRAs and have changed over time. Typically no information is provided on how each criterion and variable is weighted in the final determination of the overall credit rating.

CRAs issue their ratings in the form of letter grades. These refer to long- and short-term ratings depending on whether the evaluation is based on an horizon of more or less than 12-months. As shown in Table 1, differences in the rating scale adopted by the three main CRAs are minimal (the last column provides a broad interpretation). For reference, in the rest of this paper we adopt a rating scale similar to that currently used by Moody's (second column, Table 1). This comprises 19 grades, ranging from triple-A (Aaa), indicating the best rating quality and minimum risk, to C, which denotes obligations that are typically in default. The top ten grades, between triple-A and Baa3, are referred to as investment grade, indicating low risk obligations; the remaining 9 ratings are assigned to higher risk obligations, and thus termed as speculative grades.[6]

We use data from Moody's (2012) to highlight the following five stylized facts on the sovereign credit ratings of the EU14 countries.[7] The first stylized fact (SF1) is that the sovereign credit ratings of the EU14 countries taken as a group has been higher than those of other countries. The second stylized fact (SF2) is that the cross-section distribution of the EU14 countries sovereign credit ratings has been stable within the investment grade at least until 2010. The third stylized fact (SF3) is that sharp changes in this distribution have occurred, particularly since 2010. The fourth stylized fact (SF4) is that fluctuations in EU14 sovereign credit ratings have increased as the ratings have fallen. The fifth stylized fact (SF5) is that changes in the sovereign credit ratings of several EU 14 countries appear to be unrelated to the market's perception of the probability of sovereign default.

Table 2 provides evidence on SF1, SF2 and SF3: it reports the cross-section distribution of the sovereign credit rating of EU14 countries at selected dates between 1995 and 2012. All EU14 countries are rated as investment grade from 1990 to 2005. The share of investment-grade sovereign issuers in the EU14 group has declined since 2005. By 2012 it is still about 20 percentage points higher relative to a larger sample comprising all countries that are rated by Moody's.

[6] Gaillard (2012) provides an updated survey on the methodologies and the definitions and types of sovereign ratings currently followed by the main CRAs.

[7] There is a strong positive correlation between the sovereign ratings issued by the three main CRAs (Gaillard, 2012). Consequently, the stylized facts highlighted in this section hold regardless of the source of the sovereign ratings, whether these are taken from either Fitch Ratings, Moody's Investors Service or Standard & Poor's.

Table 1: Rating scales adopted by the three main CRAs.

	Moody's	Fitch	S&P	Credit quality
Investment grade (I.G.)	Aaa	AAA	AAA	Prime
	Aa1	AA+	AA+	High grade
	Aa2	AA	AA	
	Aa3	AA-	AA-	
	A1	A+	A+	Medium grade
	A2	A	A	
	A3	A-	A-	
	Baa1	BBB+	BBB+	
	Baa2	BBB	BBB	
	Baa3	BBB-	BBB-	
Speculative grade (S.G.)	Ba1	BB+	BB+	Speculative
	Ba2	BB	BB	
	Ba3	BB-	BB-	
	B1	B+	B+	Highly speculative
	B2	B	B	
	B3	B-	B-	
	Caa	CCC		Little prospect for recovery
	Ca	CC	CCC	
	C	C		In default
		DDD, DD, D	D	

Source: Authors' classification based on Gaillard (2012)

The share of EU14 countries in the Aaa category declined in the early 1990s and then climbed back by the early 2000s. It further declined during the latest global financial crisis, reaching the levels of the early 1990s. Until 2008 all EU14 countries were rated within the band triple A to single A; moreover, their shares in the three years before the crisis were stable. In 2009 the proportion of sovereigns rated Aa increased as a result of the downgrade of a number of triple-A countries. The downgrades in 2010 and 2011 led to a further decline in the proportion of countries rated Aaa and Aa, and an increase of the share of countries rated single-A or below. The share of speculative-grade ratings rose from 2010 to 2011 and remained stable in 2012. The distribution reached the Caa-C lower bound as a result of the Greek debt exchange proposal in February 2012, which resulted in losses for investors in excess of 70 per cent of the face value.

A time series from 1990:1 to 2012:4 of the historic credit ratings for each of the EU14 countries in Figure 1 provides evidence on SF4, the level and the volatility of the sovereign credit ratings. Four groups of countries may be identified: countries that have been rated triple-A for the whole sample period (Austria, Germany, the Netherlands and the United Kingdom, top-left panel); countries that have been rated within the top-three notches over the whole sample period (Denmark, Finland and France, top-right panel); countries that

Table 2: Distribution of historic sovereign credit ratings of EU14 countries at selected dates.

	1990	1995	2000	2005	2006-2008	2009	2010	2011	2012
Aaa	50%	36%	57%	71%	71%	64%	57%	57%	50%
Aa	36%	43%	36%	21%	21%	29%	21%	7%	14%
A	7%	14%	7%	7%	7%	7%	7%	14%	7%
Baa	7%	7%	0%	0%	0%	0%	7%	0	7%
Ba	0%	0%	0%	0%	0%	0%	7%	14%	14%
B	0%	0%	0%	0%	0%	0%	0%	0	0%
Caa-C	0%	0%	0%	0%	0%	0%	0%	7%	7%
Share of investment grade									
EU14	100%	100%	100%	100%	100%	100%	93%	79%	79%
ARC	86%	78%	59%	63%	n.a.	n.a.	61%	n.a.	60%

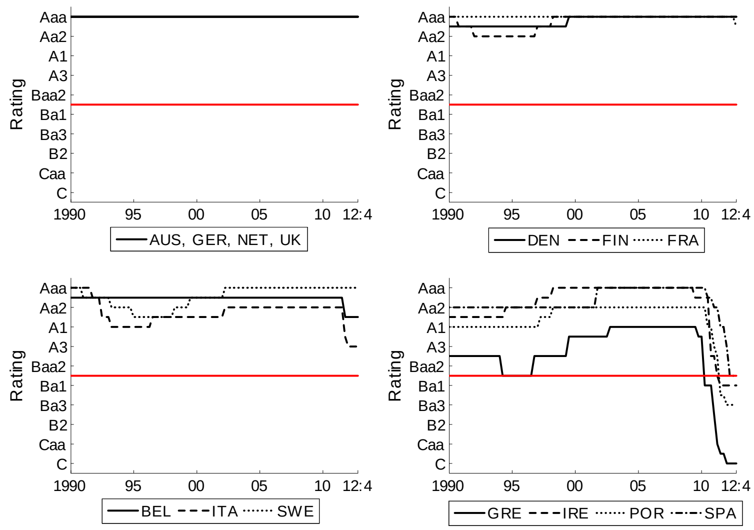
Notes: ARC=All Rated Countries in a specific year; n.a.=not available.
Source: Moody's (2012)

have always been rated within the Aaa-A range (Belgium, Italy and Sweden, bottom-left panel); and countries that have been outside the Aaa-A rating range (Greece, Ireland, Portugal and Spain, bottom-right panel). The countries in the top two panels have more stable credit ratings than those in the bottom two panels. The standard deviations of the series in each panel are, starting from the top left panel and moving clockwise, 0, 0.47, 0.93 and 2.74 respectively. As highlighted earlier, numerous revisions in the credit ratings occurred in the 1990s and from 2010.

The relation between historic credit ratings and the market perception of sovereign risk in the EU14 countries (SF5) is shown in Figure 2 which reports for the EU14 countries the daily price of credit default swaps for 5-year sovereign bonds (measured in basis points, bps) together with their sovereign credit ratings from December 2007 to March 2013.[8] Prior to 2007 there was no credit default swap market for European sovereign securities. This reflects the fact that until then government bonds in these countries were regarded as risk-free securities. For Austria, Denmark, Finland, Germany, the Netherlands, Sweden and the UK CDS prices declined after 2009 but returned to 2009 values by 2012, only to fall again afterwards. Nonetheless, the credit rating for all of these countries remained triple-A throughout. We note however that the CDS prices for these countries varied only within a moderate range compared with the other EU countries. While the CDS prices for France and the U.K. have fluctuated within a similar range, both countries have been downgraded: France in November 2012, and the U.K. in February 2013, when CDS prices on UK bonds were almost at their lowest level since 2009. The CDS prices for Belgium, France, Italy, Portugal and Spain were on an upward trend until the end of 2011 and fell afterwards. The first two countries were downgraded as their CDS prices

[8]CDS prices are taken from Thomson Reuters, accessed from Datastream in March 2013.

Figure 1: Historic sovereign credit rating of EU14 countries, 1990-2012.

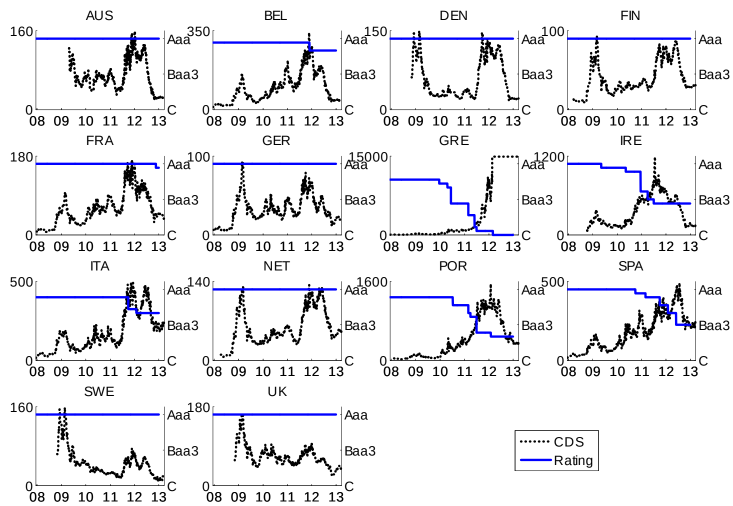


fell. Ireland received a significant downgrade as its sovereign CDS prices were increasing over the 2009-2011 period, but its credit rating was not reversed when the CDS price fell from the second half of 2011 until 2013. CDS prices on Greek bonds were traded at 50 bps between December 2007 and September 2008 then, from August 2009, they then began to increase at an almost exponential pace to reach the 400 bps mark by April 2010, the date of the first downgrade on Greek bonds.

3 Methodology

The methodology for the model-based sovereign credit rating consists in mapping the probability of sovereign default into a credit rating. The probability of default is measured by adapting for application to the government budget constraint (GBC) Black and Scholes's (1973) default formula for pricing European-style call options on underlying assets that have no intermediate payouts. Black and Scholes showed that the current value of the call depends on the risk-adjusted probability that the option will be exercised. This is determined from the projection of the current value of the asset over the maturity period, the exercise price and the asset's price volatility. Merton's (1974) formalization of this idea, when applied to government debt, entails estimating the probability that the debt-GDP ratio will exceed a given limit, or default threshold, over a specific time horizon. As we also take account of the probability of not defaulting by the end of the time horizon, we are effectively measuring the probability

Figure 2: Sovereign credit ratings and 5-year credit default swap prices for EU14 countries, 14/12/2007 - 22/03/2013.



that an American option is exercised at any time up to and including the expiry date. Default probabilities are converted into credit ratings using CRAs' records of historic long-term default experience. The implementation of the model on empirical data requires forecasts of the debt-GDP ratio and standard error of the forecast at given time horizons. The next three sub-sections describe in detail the key steps of the methodology.

3.1 Default probability

The starting point for the determination of the probability that the debt-GDP ratio will exceed a given threshold at some point over a given time horizon is the one-period GBC. Expressed as a proportion of nominal GDP, the GBC can be written as:

$$\frac{d_t}{y_t} + (1 + \rho_t)\,\frac{b_{t-1}}{y_{t-1}} = \frac{b_t}{y_t}, \tag{1}$$

where y_t is real GDP, $\frac{d_t}{y_t}$ is the primary deficit-GDP ratio, ρ_t is the discount rate - the nominal interest rate on government bonds (i_t^b) less both the inflation rate (π_t) and the growth rate of GDP (γ_t) - i.e. the output-adjusted real interest rate on government debt, and $\frac{b_t}{y_t}$ is the debt-GDP ratio. The primary deficit $\frac{d_t}{y_t}$ is defined as the difference between government expenditures on goods and services $(\frac{g_t}{y_t})$ plus transfers $(\frac{z_t}{y_t})$ - both expressed as a proportion of GDP - and the ratio of government revenues to GDP $(\frac{v_t}{y_t})$ which includes seigniorage revenues. The debt-GDP ratio in period $t+h$ is therefore

$$\frac{b_{t+h}}{y_{t+h}} = -\sum_{j=1}^{h}\left[\Pi_{s=1}^{j}\left(1+\rho_{t+s}\right)\frac{d_{t+j}}{y_{t+j}}\right] + \Pi_{s=1}^{h}\left(1+\rho_{t+s}\right)\frac{b_t}{y_t},$$

where the right-hand side is the cumulative saving generated by current and future primary surpluses from t to $t+h$ plus the interest cost of rolling-over the current debt-GDP ratio until period $t+h$.

Default is assumed to occur between periods t and $t+h$ if the expected value of the debt-GDP ratio conditional on information available in period t exceeds the threshold (debt limit) $\overline{\frac{b_{t+h}}{y_{t+h}}}$. $p_{t,t+h}$, the probability of sovereign default by period $t+h$, is the probability of not defaulting prior to year $t+h$ but defaulting in year $t+h$, and hence is given by

$$p_{t,t+h} = p_{t+h}\left(1-p_{t+h-1}\right)\left(1-p_{t+h-2}\right)...\left(1-p_{t+1}\right).$$

p_{t+h} denotes the probability of defaulting in period $t+h$ given the information available in period t, and is measured by

$$p_{t+h} = \Pr\left(\frac{b_{t+h}}{y_{t+h}} \geq \overline{\frac{b_{t+h}}{y_{t+h}}}|\Phi_t\right),$$

where $\Pr(.)$ is assumed to be the normal probability density function and Φ_t denotes information available at time t.

The default threshold $\overline{\frac{b_{t+h}}{y_{t+h}}}$ represents the amount of debt that a country will be either willing or able to repay at a specific time in the future. In practice, market analysts and investors may have in mind a debt-GDP threshold of their own, which may depend upon considerations both about a government's ability to meet its financial obligations using fiscal policy and its willingness to service its debt. We will return on how to measure and interpret the debt limit in Section 4.

The debt-GDP ratio at time $t+1$ may be decomposed into

$$\frac{b_{t+1}}{y_{t+1}} = E_t\frac{b_{t+1}}{y_{t+1}} + \xi_{t+1}$$

where $E_t\frac{b_{t+1}}{y_{t+1}}$ is the expectation of the debt-GDP ratio by the end of period $t+1$ conditional on information available in t and ξ_{t+1} is the corresponding innovation in period $t+1$. The latter may be written as

$$\xi_t = \sigma_t\varepsilon_t,$$

where $\varepsilon_t \sim i.i.d.(0,1)$. It then follows that the debt-GDP ratio for period $t+h$ may be written as

$$\begin{aligned}\frac{b_{t+h}}{y_{t+h}} &= E_t\frac{b_{t+h}}{y_{t+h}} + \eta_{t+h} \\ \eta_{t+h} &= \Sigma_{s=1}^{h}\xi_{t+s}\end{aligned}$$

where $V_t(\eta_{t+h}) = \sigma^2_{\eta,t+h} = \Sigma^h_{s=1}\sigma^2_{t+s}$ is the conditional variance of the debt-GDP ratio.

Defining

$$DD_{t+h} = \frac{E_t \frac{b_{t+h}}{y_{t+h}} - \overline{\frac{b_{t+h}}{y_{t+h}}}}{\sigma_{\eta,t+h}} \tag{2}$$

as the distance-to-default of sovereign debt, the probability of sovereign default in period $t + h$ given information in period t is

$$p_{t+h} = \Pr\left(-DD_{t+h} \leq \zeta_{t+h} | \Phi_t\right), \tag{3}$$

where

$$\zeta_{t+h} = \frac{\eta_{t+h}}{\sigma_{\eta,t+h}}.$$

The probability of default therefore increases as the gap between the expected and the threshold debt-GDP ratio ($E_t \frac{b_{t+h}}{y_{t+h}} - \overline{\frac{b_{t+h}}{y_{t+h}}}$) widens and the uncertainty surrounding the forecasts of the debt-GDP ratio (σ_{t+h}) increases. This probability changes over time as changes in the base year and in information alter the forecast of the debt-GDP ratio, its uncertainty and the debt threshold.

The probability of default in any period between t and $t+h$ (the cumulative default probability) is

$$p^c_{t,t+h} = \sum_{j=1}^{h} p_{t,t+j}, \tag{4}$$

which is calculated assuming a standard cumulative normal distribution.

Equation (2) measures the distance-to-default for given values of the debt-GDP limit, the point forecast and the standard deviation of the debt-GDP ratio at a specific time horizon. Uncertainty about these three components can be accounted for by constructing distributions of the debt-limit, the debt-GDP forecast and its conditional variance at each time horizon. The distribution of the distance-to-default can then be constructed. This can then be translated into a distribution of the probability of default using equations (3) and (4).

3.2 Mapping into credit rating

Next we require a mapping of the probability of sovereign default into a credit rating scale that includes the 19 letter-type categories (from Aaa to C) reported in the second column of Table 1. This mapping is required to make the model-based ratings directly comparable with the official ratings. Any rating scale can however be used. The starting point for constructing this mapping is Moody's (2012) record of cumulative default rates and sovereign credit rating reported in Table 3. This shows the default history of sovereign securities within specific rating categories over a 10-year horizon. Since sovereign credit ratings issued by CRAs do not entirely reflect default probabilities, it is not possible to discriminate between the Aaa and Aa ratings based solely on the history of default.

Table 3: Sovereign credit ratings and average cumulative default rates (in percentage), 1983-2012.

Year	Aaa	Aa	A	Baa	Ba	B	Caa-C
1	0	0	0	0	0.644	2.724	27.979
2	0	0	0.090	0.360	1.715	5.279	35.233
3	0	0	0.463	0.744	3.050	6.875	40.933
4	0	0	0.861	1.153	4.542	8.984	40.933
5	0	0	1.291	1.586	6.144	11.158	40.933
6	0	0	1.761	2.006	7.293	13.218	40.933
7	0	0	2.284	2.006	8.911	15.108	40.933
8	0	0	2.871	2.006	11.004	16.608	40.933
9	0	0	3.533	2.006	12.743	17.502	40.933
10	0	0	4.287	2.006	14.374	18.541	40.933
Source: Moody's (2012)							

Moreover, a default profile is available only for 7 out of the 19 categories in the second column of Table 1.

We therefore use a two-stage linear interpolation to estimate this missing information. For each year in Table 3 we derive the probability of default associated with each of the 19 categories in Table 1 by interpolating the missing observations.[9] This initial interpolation has the effect of assigning, for each year, nonzero default probabilities for ratings Aaa-Baa3 in year 1, and ratings Aaa-Aa3 in subsequent years. We then interpolate further to derive from these annual data a quarterly mapping for the whole 10-year period.[10]

The final four columns of Table 4 report the cumulative probability of default by the end of the first, fifth and tenth year, as well as the unweighted average over the whole 10-year period. The 1-year scale is used later to derive the measure of the short-term rating, while the 5-year, 10-year and average scales are used to measure long-term ratings over alternative time horizons.

3.3 Debt-GDP forecasts and volatility

The forecasts of the debt-GDP ratio and its volatility are derived from a reduced-form VAR model that allows for time variation in both parameters and volatility

[9] We assume that ratings Aa, A, Baa, Ba, B and Caa-C in Table 3 correspond respectively to Aa3, A3, Baa3, Ba3, B3 and C in Table 1 (second column). We also replace the values of 0 for A and Baa in year 1 of Table 3 with 0.09/2 and 0.36/2 respectively, i.e. half of the value in the following year.

[10] This second round of interpolation is carried out assuming that in the first year the default probability at the beginning of the first quarter is 0. We have also replaced the default probabilities at the end of the first year for Aaa ratings from 0.000*e-20 to 0.000499, as the model typically yields a nonzero default probability for Aaa ratings.

Table 4: Mapping from cumulative default probabilities to sovereign credit ratings.

	Rating		Cumulative default probability			
Category	Long-term	Short-term	1-year	5-year	10-year	average
Investment grade	Aaa	Prime - 1	0.000	0.000	0.000	0.000
	Aa1	Prime - 1	0.008	0.215	0.715	0.265
	Aa2	Prime - 1	0.015	0.430	1.429	0.529
	Aa3	Prime - 1	0.023	0.646	2.144	0.794
	A1	Prime - 1	0.030	0.861	2.858	1.058
	A2	Prime - 1/2	0.038	1.076	3.573	1.323
	A3	Prime - 1/2	0.045	1.291	4.287	1.588
	Baa1	Prime - 2	0.090	1.389	3.527	1.501
	Baa2	Prime- 2 or 3	0.135	1.488	2.766	1.415
	Baa3	Prime-3	0.180	1.586	2.006	1.329
Speculative grade	Ba1	Not Prime	0.335	3.105	6.129	3.052
	Ba2	Not Prime	0.489	4.625	10.251	4.776
	Ba3	Not Prime	0.644	6.144	14.374	6.499
	B1	Not Prime	1.337	7.815	15.763	7.962
	B2	Not Prime	2.031	9.487	17.152	9.425
	B3	Not Prime	2.724	11.158	18.541	10.887
	Caa	Not Prime	11.142	21.083	26.005	19.711
	Ca	Not Prime	19.561	31.008	33.469	28.534
	C	Not Prime	27.979	40.933	40.933	37.358

Source: Rating (www.moodys.com); Default probability (authors' calculations)

using rolling-window estimation (ROVAR model).[11] This has several advantages. First, the model can be easily estimated and updated. This is an important feature for market participants seeking to either determine or monitor their own credit-rating measure. Second, rolling-window estimation allows time variation in the parameters and volatility to be essentially data-driven; thus it can accommodate simultaneously the gradual changes in parameters and volatility that characterize the period of the great moderation (the 1990s and the 2000s) and the sudden swings observed during both the great acceleration (between the late 1970s and the early 1980s) and during the latest global financial crisis (from 2008 to 2012). Support for this approach is provided by Kapetanios et al. (2012) who find that forecasts from a rolling window VAR are not outperformed by forecasts obtained from other reduced-form models, such as the VAR with time-varying parameters and stochastic volatility of Primiceri (2005) and the Markov-switching VAR of Sims and Zha (2006). Structural models, like DSGE models, could be used to forecast the debt-GDP ratio instead of reduced form models like a VAR. If the implied restrictions are correct, then the structural

[11] Recent examples of rolling-window analyses in macroeconomics include Stock and Watson (2008), Orphanides and Wei (2012) and Canova and Ferroni (2012).

model should provide a similar forecasting performance to its associated reduced form; otherwise, the forecasts would be expected to be worse because, unlike a VAR, there is no automatic bias correction due to seeking a model with best fit. The evidence supports this assessment as forecasts from DSGE models have been found not to significantly outperform those from a VAR, particularly in the short and medium term, see Wickens (2012).

We specify an open economy ROVAR model that includes the following variables: the debt-GDP ratio ($\frac{b_t}{y_t}$), the total deficit-GDP ratio ($\frac{D_t}{y_t}$),[12] the growth rate real GDP (γ_t), the inflation rate (π_t), a short-term interest rate (r_t^s), a long-term interest rate (r_t^l), the real exchange rate (e_t), the ratio of the current account to GDP ($\frac{x_t}{y_t}$) and the oil-price inflation rate (π_t^o). Quarterly observations for each variable are available from 1977:2 to 2012:4 for Portugal, and from 1975:2 to 2012:4 for all other countries. The data for Germany prior 1991 refer to West Germany alone. Appendix A.1 provides details. The first four variables capture the behavior of the fiscal and the domestic private sectors. They also allow the model to implicitly satisfy the GBC. The short- and long-term interest rates capture the links between the debt-GDP ratio, monetary policy and the term structure. The last three variables reflect the impact of the external sector (the exchange rate and the current account balance) and global economic factors (the oil-price inflation rate) on the domestic macroeconomic and fiscal outlooks. Reinhart and Rogoff (2008, pp.6) document that "peaks and troughs in commodity price cycles appear to be leading indicators of peaks and troughs in the capital flow cycle, with troughs typically resulting in multiple defaults". The variables included in the ROVAR give a description of open economies typical of the empirical literature on fiscal shocks and business cycle fluctuations that is based on reduced-form models; see for example Fatas and Mihov (2001), Canzonieri et al. (2002), Chung and Leeper (2007).

The ROVAR is specified with a constant and one lag in each equation; it is estimated with OLS using a rolling-window sample of 30 quarters for all countries. We generate forecasts of the distribution of the debt-GDP ratio over an horizon of 40 quarters from 1995:4 to 2012:4. The forecast variance is measured from the covariance matrix of the h-period ahead forecast error. None of the average adjusted R-squareds for the debt-GDP equation for the rolling samples is below 97 per cent, which suggests that the parsimonious specification of the ROVAR model is sufficient to provide a good description of the data generating process for the debt-GDP ratio.[13] Figure 3 shows actual debt-GDP ratio for the EU14 countries from 1995:1 to 2012:4 together with the estimated standard deviations of the 1-period ahead forecast errors from the ROVAR which we draw on when interpreting the results in Section 5. Two features are of particular relevance. First, in all countries volatility is positively related to the level of the debt-GDP ratio for most of the sample period and, in particular, from the second half of the 2000s. This co-movement between the level and the volatility

[12] From equation (1), the total deficit is $\frac{D_t}{y_t} = \frac{d_t}{y_t} + \rho_t \frac{b_{t-1}}{y_{t-1}}$.

[13] For reasons of space we do not report descriptive statistics of the data and the ROVAR estimates. These are available upon request from the authors.

of the debt-GDP ratio has important implications for the measurement of the default probability and the sovereign credit rating. Uncertainty, measured by the standard deviation of the ROVAR forecast error, increases over the forecasting horizon. Equation (2) implies that this has the effect of increasing the default probability, in turn reducing the credit rating. If the actual debt-GDP ratio has an increasing (declining) trend, then the ROVAR typically forecasts an increasing (decreasing) debt-GDP ratio and equation (2) a higher probability of default. This is compounded by the effect of uncertainty over the forecasting horizon. Second, in all countries, except Sweden, the debt-GDP ratio starts to increase from the second half of the 2000s. For 10 countries the starting date of the increase in the debt-GDP ratio is the year 2007; for 6 of these it is 2007:2. This common pattern clearly marks the beginning of deterioration in the EU fiscal stances 4 quarters before the collapse of Lehman in September 2008. This deterioration in European fiscal stances is connected with the conduct of U.S. and European monetary policy in the late 1990s and early 2000s, which was characterized by low policy rates in both.[14] By 2003:4 the federal funds rate reached its lowest value (1 per cent) since 1960. It then began to increase, peaking at about 5.25 per cent by 2007:1. This triggered the burst of the U.S. housing bubble (between 2005 and 2006) and an increase in rates of interest across the world. In Europe, the short-term rate reached its lowest value, about 2 per cent, in 2005 and then increased to peak at about 5 per cent in 2008:2. The increase in interest rates had a direct negative effect on the public finances of EU countries by raising the cost of public borrowing. It also had an indirect negative effect as in several countries it burst a house-price bubble and led to a fall in output and an increase in unemployment which reduced tax revenues and increased public expenditures. This interpretation would suggest that the eurozone sovereign debt crisis was ultimately a negative spillover of international monetary policy.

4 Debt limits

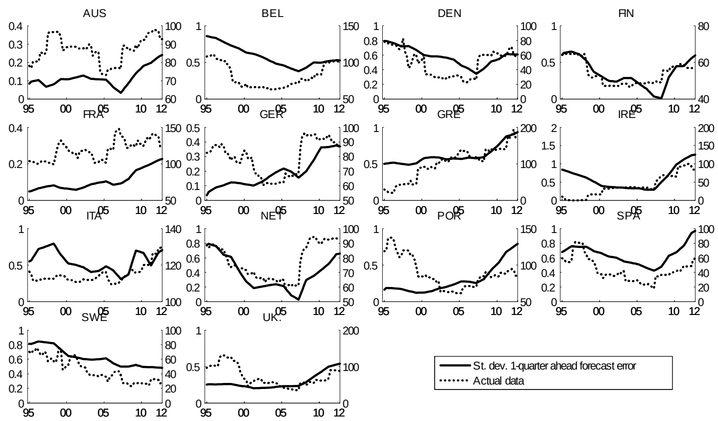
4.1 Theory

Measuring the value of the debt-GDP ratio above which a government is expected to default is neither straightforward nor uncontroversial. Market analysts and investors may have in mind a debt-GDP threshold of their own, which may depend on subjective considerations about a government's ability and willingness to meet its financial obligations.

The empirical literature on debt-GDP ratios at times of default can be employed to construct rule-of-thumb estimates of the debt limit. Burnside's (2005) review of this literature points out that "safe" debt-GDP levels for countries that have experienced a series of defaults are much lower than those of industrialized countries. They also vary over time. This suggests that a meaningful cross-

[14] Taylor (2010) provides an insightful reflection on the conduct and implications of U.S. monetary policy in the period leading up to the crisis.

Figure 3: Debt-GDP ratio in EU14 countries, 1995:1-2012:4: actual observation (solid line) and standard deviation of 1-period ahead forecast error (dotted line) from ROVAR model.



country comparison of sovereign credit ratings should be based on a measure of the debt limit that is state and time dependent.

There are a number of explanations in the theoretical literature about why sovereigns choose to service their debt rather than default, for example, the risk of exclusion from the capital market (Eaton and Gersovitz, 1981), incurring economic sanctions (Sachs, 1984), or loosing sovereign reputation (Eaton and Fernandez, 1995). The main problem with these explanations is that the predicted level of government debt at which sovereign default is likely to occur is low relative to the debt levels observed in developed countries (Arellano, 2008). Models of liquidity crises - for example, Cole and Kehoe (2000) - can be used to derive debt-GDP thresholds. Above these thresholds default is however undetermined as it depends on whether a country can still avoid a liquidity crisis. Broner, Martin and Ventura (2010) have recently extended this theoretical literature by considering the role of secondary markets in determining sovereign default events. More recently, a new literature has emerged on the determination of default thresholds based on using dynamic macroeconomic models, see Davig, Leeper and Walker (2010, 2011) and Bi (2011). The assumption in this approach is that the willingness of a government to repay its debt depends on whether it is able to generate the required financial savings. Excluded are considerations of whether or not generating these savings are politically feasible. This literature focuses on the ability of governments to raise revenue from unanticipated changes in distortionary taxes that are bounded above due to the Laffer effect, given the market expectation of future government expenditures. As a result, a government may be unable to generate enough revenue to finance its debt, particularly when debt is high. Default therefore occurs endogenously in the model when the equilibrium level of debt exceeds its feasible upper bound. This is referred to as the fiscal limit.

We extend this literature in four ways. First, to compute the Laffer curves we employ an open-economy rather than a closed-economy model as used by Trabandt and Uhlig (2011). Second, we consider distortionary taxation on income from labor, capital and consumption rather than labor alone. Third, we show that the fiscal limit is a special case of a broader range of debt limits that can be derived from DSGE macroeconomic models. Fourth, we determine a time-series of these debt limits in order to evaluate how and why they have changed over time. For each country, the model of the economy includes four sectors: households, firms, the government and the rest of the world. The analytical framework is described by the following equations:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, 1-n_t\right); \tag{5}$$

$$(1+\tau_t^c)c_t + k_t + b_t^D + s_t f_t = (1-\tau_t^n)w_t n_t + \left(r_t^k - \delta\right)\left(1-\tau_t^k\right)k_{t-1} + \tag{6}$$

$$+k_{t-1} + \left(1-\xi_t^D\right) r_t b_{t-1}^D + \left(1-\Xi_t^D\right) b_{t-1}^D + z_t + \left(1+r_t^*\right) s_t f_{t-1};$$

$$c_t = \left[\phi\left(c_t^H\right)^{1-\frac{1}{\eta}} + (1-\phi)\left(c_t^F\right)^{1-\frac{1}{\eta}}\right]^{\frac{1}{1-\frac{1}{\eta}}}; \tag{7}$$

$$y_t = k_t^{\alpha}\left(A_t n_t\right)^{1-\alpha}; \tag{8}$$

$$g_t + \left(1-\xi_t^D\right) r_t b_{t-1}^D + \left(1-\Xi_t^D\right) b_{t-1}^D + \left(1-\xi_t^F\right) r_t b_{t-1}^F + \tag{9}$$

$$+\left(1-\Xi_t^F\right) b_{t-1}^F + z_t = \tau_t^c c_t + \tau_t^n w_t n_t + \tau_t^k \left(r_t^k - \delta\right) k_{t-1} + b_t^D + b_t^F;$$

$$s_t f_t - b_t^F = x_t + \left(1+r_t^*\right) s_t f_{t-1} - \left(1+r_t\right) b_{t-1}^F; \tag{10}$$

$$y_t = c_t + g_t + k_t - (1-\delta) k_{t-1} + x_t. \tag{11}$$

Households derive utility from total consumption c_t and leisure $1-n_t$, and seek to maximize their lifetime utility in equation (5); where E_0 denotes mathematical expectation conditioned on time 0 information, $\beta \in (0,1)$ is the household discount factor, $u(.)$ is a twice continuously differentiable, increasing, strictly concave utility function and n_t denotes the supply of labor. Household maximization is subject to the budget constraint, equation (6), in which k_t, b_t^D, f_t, w_t, s_t, r_t^k, r_t, r_t^*, z_t, δ, τ_t^c, τ_t^n and τ_t^k respectively denote physical capital, government bonds held by domestic households, real net foreign assets denominated in foreign currency, the real wage, the real exchange rate (defined as the home currency per unit of foreign currency), the real rate of return from capital, the domestic real rate of return on bonds, the real rate of return on foreign assets, government transfers, the rate of physical depreciation, the tax rates on consumption, labor income and net income from capital, $r_t^k - \delta$. Parameters $\xi_t^D \in (0,1)$ and $\Xi_t^D \in (0,1)$ denote shares of government bonds interest and principal respectively lost by households due to default. These variables are treated as exogenous as the aim is to derive stationary equilibrium solutions of the debt limits that account for the default risk, rather than to identify an endogenous transmission mechanism linking default risk to interest rates as, for example, in Bi (2011). There is imperfect substitutability between home and foreign goods. Total consumption is assumed to satisfy the CES function described in equation (7); c_t^H, c_t^F, ϕ and η denote goods purchased domestically, goods purchased from abroad, the relative expenditure weight on domestic and foreign goods, and the elasticity of substitution between domestic and foreign goods respectively. Output is generated by the labor-augmenting Cobb-Douglas production function (8), where A_t denotes technological progress and α is the income share of capital. Equation (9) describes the government budget constraint, where g_t is government expenditure on goods and services, and b_t^F is government debt held abroad and denominated in domestic currency. The variables $\xi_t^F \in (0,1)$ and $\Xi_t^F \in (0,1)$ capture the probability of the government defaulting on the interest and the principal repayments to non-residents holding domestic bonds. In order to allow the reconciliation of total revenue and tax revenue in the data, z_t is measured as gross transfers net of any source of government revenue other than taxation. The balance of payments and the national income identity are described by equations (10) and (11) respectively, where x_t denotes net foreign trade expressed in domestic currency.

Appendix B shows that under the utility function

$$u(c_t, 1-n_t) = \log c_t + \psi \log(1-n_t) \tag{12}$$

four alternative version of the maximum borrowing capacity (debt limit) of an economy can be computed from the stationary equilibrium solution of the model. These are

$$\frac{b}{y}^{IGBCL} = \frac{1}{r}\left\{\begin{array}{c} \tau^c\chi\left(\frac{1}{\varphi k}-1\right)+\tau^n(1-\alpha) \\ +\tau^k\alpha\left[1-\delta\left(\frac{\beta^{-1}-1}{1-\tau^k}+\delta\right)^{-1}\right]-\frac{g}{y}-\frac{z}{y} \end{array}\right\} \tag{13}$$

$$\frac{b}{y}^{NDL} = \frac{1}{r}\left\{\begin{array}{c} \tau^c\chi\left(\frac{1}{\varphi k}-1\right)+\tau^n(1-\alpha) \\ +\tau^k\alpha\left[1-\delta\left(\frac{\beta^{-1}-1}{1-\tau^k}+\delta\right)^{-1}\right] \end{array}\right\} \tag{14}$$

$$\frac{b}{y}^{FL} = \frac{1}{r}\left\{\begin{array}{c} \tau^c\chi\left(\frac{1}{\varphi k}-1\right)+\tau^{n,\max}(1-\alpha) \\ +\tau^{k,\max}\alpha\left[1-\delta\left(\frac{\beta^{-1}-1}{1-\tau^k}+\delta\right)^{-1}\right]-\frac{g}{y}-\frac{z}{y} \end{array}\right\} \tag{15}$$

$$\frac{b}{y}^{MDL} = \frac{1}{r}\left\{\begin{array}{c} \tau^c\chi\left(\frac{1}{\varphi k}-1\right)+\tau^{n,\max}(1-\alpha) \\ +\tau^{k,\max}\alpha\left[1-\delta\left(\frac{\beta^{-1}-1}{1-\tau^k}+\delta\right)^{-1}\right] \end{array}\right\} \tag{16}$$

where

$$r = \frac{r^* + \Xi^D}{1-\xi^D} \tag{17}$$

$\chi = \frac{(1-\tau^N)}{\psi(1+\tau^C)}(1-\alpha)$, $\varphi = \left[\frac{\beta^{-1}-1+\delta(1-\tau^k)}{\alpha A^{1-\alpha}(1-\tau^k)}\right]^{\frac{1}{1-\alpha}}$, $k = \frac{\mu+(1+\tau^c)(g+x)}{[(1+\tau^c)\Omega+\mu\varphi]}$, $\mu = \frac{1}{\psi}(1-\tau^n)(1-\alpha)A^{1-\alpha}\varphi^{-\alpha}$ and $\Omega = (A\varphi)^{1-\alpha} - \delta$. All four solutions are non-linear in the three tax rates τ_t^c, τ_t^n and τ_t^k.

Equation (13) is the stationary equilibrium solution for the debt-GDP ratio under anticipated policy. The existence of an equilibrium solution implies that the intertemporal GBC is satisfied and that a government cannot roll over its liabilities forever (the No-Ponzi game condition). It also implies that governments can borrow at a rate that allows an equilibrium to exist. The resulting stationary equilibrium debt-GDP ratio must be equal to the market expectation of discounted stationary equilibrium future primary surpluses. In this respect equation (13) is a debt-GDP limit identifying a government's borrowing capacity based on the market's anticipation of the future evolution of fiscal and monetary policy. We will refer to this measure of the debt-GDP limit as IGBCL.

The other three debt-GDP limits are derived by considering the potential maximum impact of unanticipated changes in fiscal policy. These are, by definition, unpredictable. Nonetheless, to the extent that government revenues and expenditures are bounded (from above and below respectively) market participants would be able to determine the maximum potential impact of unexpected changes in fiscal policy on the stationary equilibrium debt-GDP ratio.

Equation (14) measures the potential effect on the borrowing capacity due to cutting government expenditure to the minimum. As government expenditure is bounded from below, it is non-negative. The debt limit in equation (14) is obtained by imposing the additional constraints in equation (13) that $\frac{g}{y} = \frac{z}{y} = 0$. This adapts to government policy Aiyagari's (1994) natural debt limit. We therefore refer to this debt limit as the NDL. Having, in effect, eliminated government expenditures, the NDL limit precludes a government from being able to finance higher debt levels from unanticipated reductions in expenditure; instead it must use unanticipated increases in taxation or changes in monetary policy.

Equation (15) measures the maximum potential effect on the debt limit of an increase in tax rates in an economy with distortionary taxation where government revenue is bounded from above due to the Laffer effect. This is obtained by replacing τ^n and τ^k in equation (13) with the tax rates $\tau^{n,\max}$ and $\tau^{k,\max}$ that maximize tax revenues from labor and capital respectively. Since there is no Laffer effect associated with the distortionary taxation of consumption in conventional real business cycle models, see for example Trabandt and Uhlig (2011), the tax rate on consumption is kept at its anticipated equilibrium value. This measure of the debt limit is, in effect, an adaptation to an open economy (with distortionary taxation on income from labor, capital and consumption) of the fiscal limit derived by Davig, Leeper and Walker (2010, 2011) and Bi (2011). We refer to this debt-GDP limit by FL. It identifies the point where the government no longer has the ability to increase its borrowing capacity by unanticipated changes in tax policy. Nonetheless, it could still either change its expenditure policy or use monetary policy, or both, see for example Cochrane

(2011).

Equation (16) measures the maximum stationary equilibrium value of the debt-GDP ratio, obtained by imposing on equation (13) the conditions applied to both the NDL and the FL. We refer to this as the maximum debt limit, MDL. At the MDL, a government can no longer use unanticipated changes in fiscal policy to finance additional debt and so would then need to resort to monetary policy.

This benchmark model excludes the possibility that a government could inflate away its debt obligations. There are two reasons for this. First, the fiscal-consolidation strategies to reduce the budget deficits in advanced economies that have been proposed by the IMF explicitly exclude inflation (seigniorage revenue) as a policy instrument, see Cottarelli (2010). Second, in the euro zone, monetary policy has been delegated to the ECB which has set a low inflation target. This leaves little scope for a member government to raise unanticipated seigniorage revenues.[15]

Although, like a tax on consumption, there is no Laffer effect in the above model for an inflation tax, it would be possible to respecify the money demand function in the model to produce a Laffer effect for inflation. This can be achieved by replacing the cash-in-advance constraint by an interest elastic money demand function. An unanticipated increase in inflation would lead to an increase in the nominal rate of interest and a contraction in the demand for real money balances thereby producing a Laffer effect. This would result in a de facto default on non-inflation-indexed bonds and would be inconsistent with the notion of a maximum repayment capacity that is implicit in the debt limit.

Equation (17) gives the stationary equilibrium, country-specific, default-adjusted rate of return on government bonds. We calibrate this by the spread in average rates of return on government bonds across countries (see Step 6 in Appendix C). A time-varying and country-specific risk premium is also (implicitly) accounted for in the ROVAR forecast of the debt-GDP ratio.

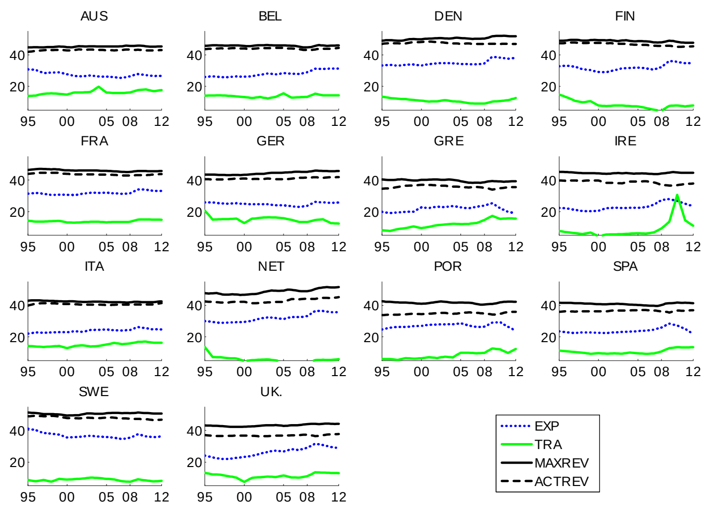
4.2 Numerical evaluation

We derive the stationary equilibrium solution of the four debt-GDP limits using the nonlinear Monte Carlo Markov Chain algorithm for solving rational expectations models of Judd (1998).[16] This is consistent with the nonlinear solution method of Coleman (1991) that was recently employed by Bi (2011) for computing the FL for a number of advanced countries. The algorithm provides time-varying and state-dependent distributions of each of the four debt-GDP

[15] The effects of anticipated inflation are implicitly accounted for in the ROVAR forecast of the debt-GDP ratio.

[16] In principle, the model solution can be computed using a standard perturbation approach, for example, by taking a local approximation based on a Taylor series expansion. Perturbation methods, however, are local approximations reliable only when disturbances represent small deviations from the steady state. They are not, therefore, suitable for evaluating large temporary deviations of the debt-GDP ratio from its stationary equilibrium. Furthermore, the solution of a rational expectations model obtained with perturbation methods can only be implemented using stationary data which is not a feature of recent macroeconomic data.

Figure 4: Components of the theory-based debt limits for EU14 countries, 1995:4-2012:4. All variables are as a proportion to GDP.



limits. These are obtained by calibrating the model using rolling-window means of the ratio of government expenditures to GDP, the ratio of transfers to GDP, the shocks to technological progress, the actual and the maximum tax rates. Appendix C describes the algorithm in detail.

The key variables contributing to changes in the four debt-GDP limits for the EU14 countries over the period 1995:4-2012:4 are shown in Figure 4. They are the ratios of government expenditure in goods and services as a proportion to GDP ($\frac{g}{y}$, denoted as EXP), the transfers-GDP ratio ($\frac{z}{y}$, denoted as TRA), the actual revenue-GDP ratio (ACTREV) and maximum revenue-GDP ratio (MAXREV). For nearly all countries the gap between ACTREV and MAXREV is small. This suggests that there is little scope for raising tax revenues and that an expansion of borrowing capacity may require expenditure cuts. Given that tax revenues are usually much more strongly positively correlated with GDP than expenditures, an increase in GDP may be sufficient to achieve this.

Increases in EXP and TRA would reduce IGBCL and FL with no effect on FL and MDL; an increase in ACTREV would increase IGBCL and NDL, with no effect on the other two limits; and an increase in MAXREV increases FL and MDL, with no effect on IGBCL and NDL. The two revenue series ACTREV and MAXREV are fairly stable over the 1995-2012 period; transfers and expenditures fluctuate more. ACTREV for Denmark, Finland and Sweden is on average about 50 per cent; this is significantly higher than for the other countries. Government expenditures increase significantly in Greece, Ireland, Portugal, Spain

Table 5: Average values of the debt limits for EU14 countries 1995:4-2012:4.

	IGBCL	FL	NDL	MDL
AUS	1.85	2.35	11.38	11.88
BEL	2.20	2.48	11.06	11.34
DEN	3.36	3.85	12.98	13.47
FIN	3.39	3.72	12.40	12.73
FRA	1.76	2.25	11.65	12.14
GER	2.17	2.89	11.50	12.21
GRE	0.37	0.89	4.05	4.57
IRE	2.38	3.47	7.97	9.05
ITA	1.17	1.50	8.11	8.44
NET	3.17	4.13	12.07	13.03
POR	1.17	2.48	6.34	7.65
SPA	1.26	2.20	7.72	8.66
SWE	2.57	2.98	12.52	12.93
UK	1.28	2.59	8.47	9.78
EU14				
Mean	2.01	2.70	9.87	10.56
St. dev.	0.24	0.20	0.17	0.12

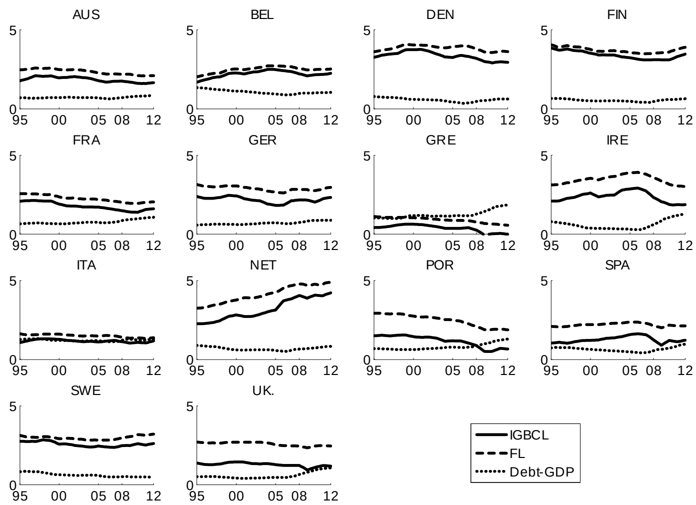
and the U.K. until the end of 2009; from about 2010 they begin to fall as a result of fiscal consolidation plans undertaken in all five countries.

The average values of the four debt limits reported in Table 5 show significant differences. NDL and MDL, which are based on zero government expenditures, are much higher than IGBCL and FL, which are based on expected expenditures and are therefore more realistic. While NDL and MDL imply overall average debt limits of 9.87 and 10.56 times GDP, respectively, IGBCL and FL imply debt limits of 2.01 and 2.70. NDL and MDL also fluctuate less due to eliminating expenditures. The difference between IGBCL and FL shows the effects of maximizing tax revenues.

Looking at country differences, the debt limits for the Scandinavian countries and the Netherlands are the highest. They also have the highest tax revenues as a proportion of GDP. The countries with the lowest debt limits are Greece, Ireland, Portugal, Spain and the UK. These are the countries most affected by the latest financial crisis.

Figure 5 shows how the IGBCL and FL and the historic debt-GDP ratios have evolved over the period 1995:4 to 2012:4 for the EU14 countries. These data give useful information about their fiscal stances. The fiscal stance is sustainable, in the sense that government are not over-borrowing under anticipated policy, if the debt-GDP ratio lies below the IGBCL as this implies that expected future fiscal surpluses are sufficient to repay existing debt, see Polito and Wickens (2011a). A debt-GDP ratio below FL implies that a government may still be solvent by implementing revenue-maximizing taxation. The historic debt-GDP ratio are below the IGBCL and the FL for all EU14 countries except Greece

Figure 5: IGBCL, FL and debt-GDP ratio in EU14 countries, 1995:4 - 2012:4.

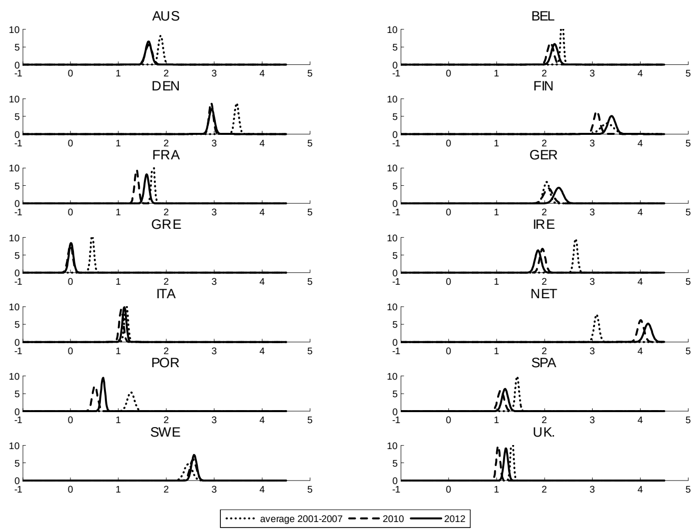


(throughout the sample period) and Portugal (where its IGBCL lies below its debt-GDP ratio from 2008). The debt-GDP ratio is almost the same as the IGBCL for Italy throughout the sample period, and for Spain and the UK from 2008; for France and Ireland they have been converging. This shows the impact of the financial crisis on their fiscal stances. For most countries the two debt limits do not fluctuate greatly. The main exception is the Netherlands where IGBCL and FL have increased over time. The gap between IGBCL and FL has also been fairly stable and is quite small for Austria, Belgium, Denmark, Finland, France, Greece, Italy and Sweden. The IGBCL and FL of Greece and Portugal have fallen steadily over the sample period, while for Ireland they have fallen since 2008.

The debt limits are estimates and so are random variables. They also have time-varying distributions. Figures 6 and 7 show how the distributions of IGBCL and the FL have changed over the sample period. The dotted line denotes the average probability density functions (PDF) from 2001 to 2007; the dashed line is the PDF in 2010 and the solid line is the PDF in 2012.

Except for Sweden and the Netherlands, between 2007 and 2010 the distributions of both debt limits have shifted to the left, showing a lower borrowing capacity due to increased government expenditures. The PDFs of the IGBCL for Finland, France, Germany, the Netherlands, Portugal and the UK have shifted to the right since 2010 as a result of fiscal consolidation; only Ireland's has shifted to the left. The PDFs of FL have all either shifted to the right or remained the same since 2010. The distributions of both debt limits have

Figure 6: State-dependent probability density function of the IGBCL of EU14 countries at selected dates.



shifted to the left for both Greece and Ireland.

5 Model-based ratings for EU14 countries

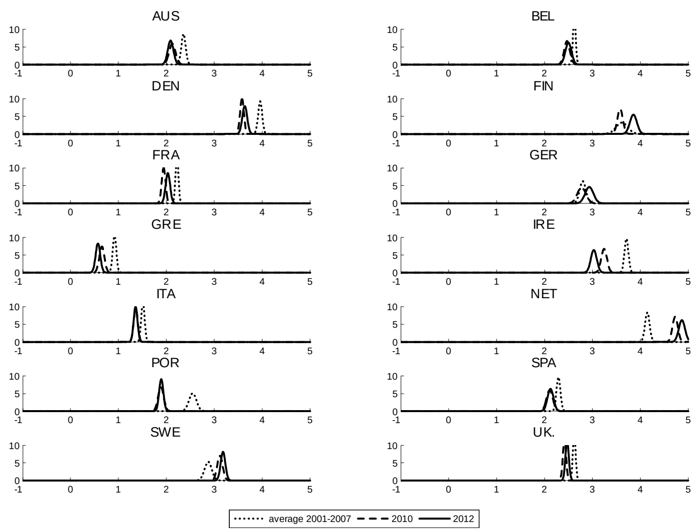
5.1 Main findings

In this section we report the model-based estimates of the credit ratings of the EU14 countries which are based exclusively on an assessment of the financial ability of governments to use their fiscal instruments to meet their outstanding debt obligations.[17] Differences with the credit ratings issued by the CRAs can be attributed primarily to the non-fiscal factors that they include in their judgements. As previously noted, these may include, among other considerations, the willingness of a government to repay its debt, the political feasibility of implementing required fiscal changes, the possibility of financing debt through domestic monetary policy and the likelihood of receiving international bailouts.

Figure 8 shows the model-based credit ratings for the EU14 countries for

[17] We report a smoothed version of the model-based credit rating determined as follows: in the first period of the sample the reported credit rating is set equal to the initial credit rating; if the new initial credit rating (from the second period onwards) is the same as the previous quarter's initial rating, the new reported rating is set equal to the rating reported in the previous quarter; if the new initial credit rating is higher (lower) than the previous period's initial rating then the reported credit rating is upgraded (downgraded) by one notch. Polito and Wickens (2012b) explain this in detail and provide examples based on U.S. data.

Figure 7: State-dependent probability density function of the FL of EU14 countries at selected dates.

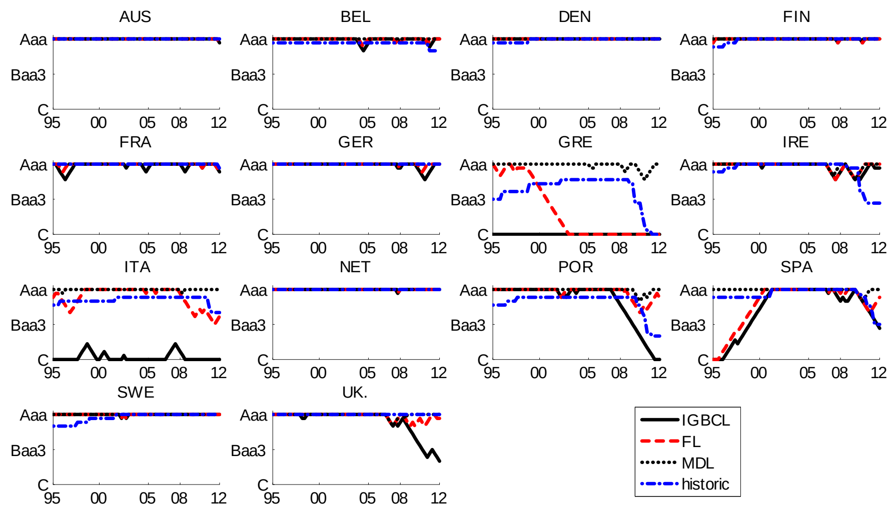


the period 1995:4-2012:4. These are based on debt-GDP forecasts and default probabilities at the 5-year horizon and three debt limits: IGBCL (dashed line), FL (solid line) and MDL (dotted line); for reference, the historic sovereign credit rating is also reported (dashed-dotted line). The model-based credit ratings differ across countries. They are also affected by the choice of debt limit. In general, downgrades are more likely, and last longer, using the IGBCL than the FL limit. MDL, the highest debt limit, generates an implausible triple-A credit rating for most countries for most of the sample period. Even using the highest debt limit, however, the model-based credit rating downgrades Greece, Ireland, Portugal and Spain from 2007 onwards.

Differences between the historic and the model-based credit ratings depend on the country.[18] Denmark is the only country with a triple-A credit rating for the whole sample period. In contrast, the model-based credit ratings for Greece, Ireland, Italy, Portugal Spain and the U.K. vary considerably depending on the definition of the debt limit. For the other countries the credit ratings show little sensitivity to the choice of debt limit, despite minor short-term downgrades from triple-A, mainly occurring after 2005, and when using the IGBCL.

[18] We have also derived the credit ratings based on the other three forecasting horizons for the computation of the cumulative default probability, as in Table 4. This shows that downgrades occur more frequently and for prolonged periods the longer is the forecasting horizon. Using the average default probability yields results similar to the 10-year horizon. These results, which are not reported in the main text for reasons of space, are available upon request.

Figure 8: Model-based (5-year horizon) and historic credit ratings in EU14 countries, 1995:4-2012:4.

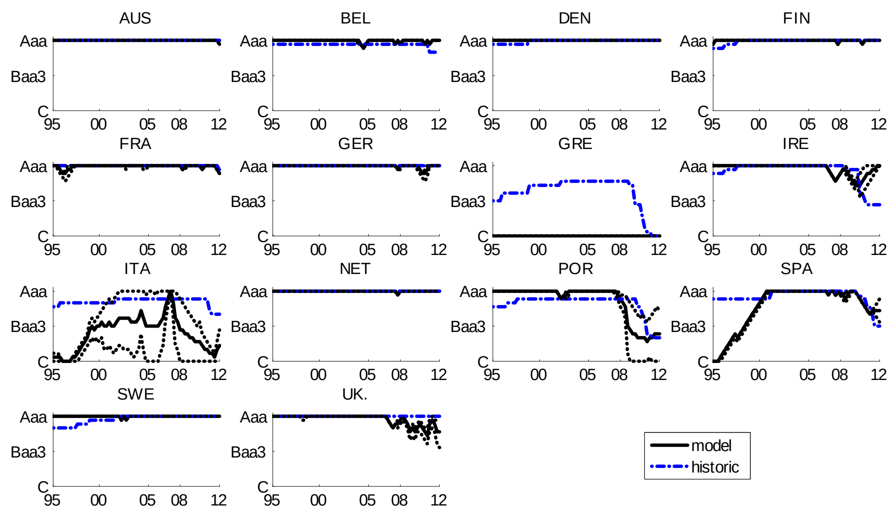


A clearer idea of the effects on country credit ratings of uncertainty about the appropriate measure of the debt limit may be obtained by dividing the difference between the two limits into one percentage point increments. Credit ratings may then be constructed at each point in the grid to form a distribution of credit ratings. The median, 16th and 84th percentile values of the rating distribution obtained using the IGBCL as lower bound and the FL as upper bound are shown in Figure 9. The letter grades corresponding to the median values for each country are reported in Table 8 in Appendix D.

Three groups of countries may be identified. In the first group are Austria, Belgium, Denmark, Finland, France, Germany, the Netherlands and Sweden. Their model-based credit ratings are close to their historic rating in being triple-A for most of the 1995-2012 period. Downgrades from the triple-A rating occur for short periods and do not exceed 1 or 2 notches. In the second group are Ireland and the U.K. Their downgrades for the second half of the 2000s anticipate the downgrades observed in the historic ratings. In both countries the model-based credit ratings begin to improve from about 2011 onwards.[19] The third group consists of Greece, Italy, Spain and Portugal, each of which has a very different rating profile. For several years they also have historic credit ratings that are significantly different from their model-based ratings. For Por-

[19] The recovery of the model measure of the credit rating towards the triple-A mark for Ireland and the U.K. during 2011-2012 is also driven by the fact that the forecasts of the debt-GDP ratio in these three countries are quickly mean reverting.

Figure 9: Model-based (5-year ahead) and historic credit ratings for EU14 countries, 1995:4 - 2012:4. Debt limit ranges from IGBCL to FL. Dotted lines denote confidence bands.



tugal the model-based credit rating is higher than the historic rating until 2008 when it falls more sharply than the historic rating before stabilizing at a similar level. For Italy the historic credit rating has been significantly higher than the model-based rating during the second half of the 1990s and from 2008. During 2008-2012 the model-based rating for Italy has fallen much more sharply than the historic rating. For Spain, the model-based rating is significantly lower than the historic rating until the early 2000s. The two then move together until the beginning of the second half of the 2000s when the model-based rating starts to downgrade. For Greece the historic credit rating is much higher than the model-based rating for the whole period. The C-grade rating throughout reflects the finding in Figure 4 that Greece's debt-GDP ratio has been below the FL debt limit over the same period.[20]

Table 6 reports sample averages and the number of rate changes for both the model-based and the historic credit ratings. The average model-based rating is lower than the historic by more than 2 notches for the whole sample period for only Greece, Italy and Spain; this happens for Ireland, Portugal and the U.K. during the period 2008-2012. In addition, for the model-based credit

[20] In a separate exercise we have examined the effects of uncertainty about the debt-GDP forecasts by recalculating the credit rating using bootstrapped forecasts. This caused a widening of the confidence bands in Figure 9. The results are not reported here for reasons of space, but are available upon request from the authors.

Table 6: Model-based and historic sovereign credit rating of EU14 countries, summary statistics.

	Average credit rating			
	Model		Historic	
	1995-2012	2008-2012	1995-2012	2008-2012
AUS	Aaa	Aaa	Aaa	Aaa
BEL	Aaa	Aaa	Aa1	Aa1
DEN	Aaa	Aaa	Aaa	Aaa
FIN	Aaa	Aaa	Aaa	Aaa
FRA	Aaa	Aaa	Aaa	Aaa
GER	Aaa	Aaa	Aaa	Aaa
GRE	C	C	Baa1	Ba1
IRE	Aa1	Aa2	Aa1	A1
ITA	Ba2	Ba3	Aa3	Aa3
NET	Aaa	Aaa	Aaa	Aaa
POR	Aa3	Baa2	Aa3	A3
SPA	A1	Aa2	Aa1	Aa2
SWE	Aaa	Aaa	Aa1	Aaa
UK	Aa1	Aa3	Aaa	Aaa
	Credit rating changes			
	Model		Historic	
	1995-2012	2008-2012	1995-2012	2008-2012
Total	168	98	40	24
Downgrades	82	61	24	24

Notes: Authors' calculations based on data in Figure 12.

ratings revisions are twice as frequent as for the historic ratings, though a similar proportion (about 60 per cent) of revisions occurred over the period 2008-2012. Whether these differences reflect a systematic overstatement of credit ratings by the CRAs or are the result of including factors additional to those associated with the fiscal position is unclear.

5.2 Stylized facts revisited

The five stylized facts about EU14 historic credit ratings identified in Section 2 may be revisited in the light of the model-based credit ratings. The cross-country distributions of the model-based credit ratings of the EU14 countries at selected dates over the period 1995-2012 are reported in Table 7. The table shows that SF1 still holds when using the model-based credit rating: the share of EU14 countries rated investment grade is still higher than for other countries (see Table 2). SF2 no longer holds as the distribution does not lie entirely within the investment grade. Instead, it is bimodal during the pre-crisis period. SF3, which is related to changes in the mix of grades, also appears to hold no longer. Previously we noted that significant changes in the distribution of the historic

Table 7: Distribution of the model-based sovereign credit rating of EU14 countries at selected dates based on the median value rating when the debt limit ranges between FL and IGBCL.

	1995	2000	2005	2006	2007	2008	2009	2010	2011	2012
Aaa	71%	79%	86%	86%	79%	50%	64%	50%	50%	50%
Aa	7%	7%	0%	0%	14%	36%	7%	21%	21%	14%
A	0%	0%	0%	0%	0%	0%	7%	7%	7%	14%
Baa	0%	7%	7%	7%	0%	0%	0%	0%	0%	0%
Ba	0%	0%	0%	0%	0%	7%	14%	7%	7%	7%
B	0%	0%	0%	0%	0%	0%	0%	7%	0%	7%
Caa-C	21%	7%	7%	7%	7%	7%	7%	7%	14%	7%
IG	79%	93%	93%	93%	93%	86%	79%	79%	79%	79%

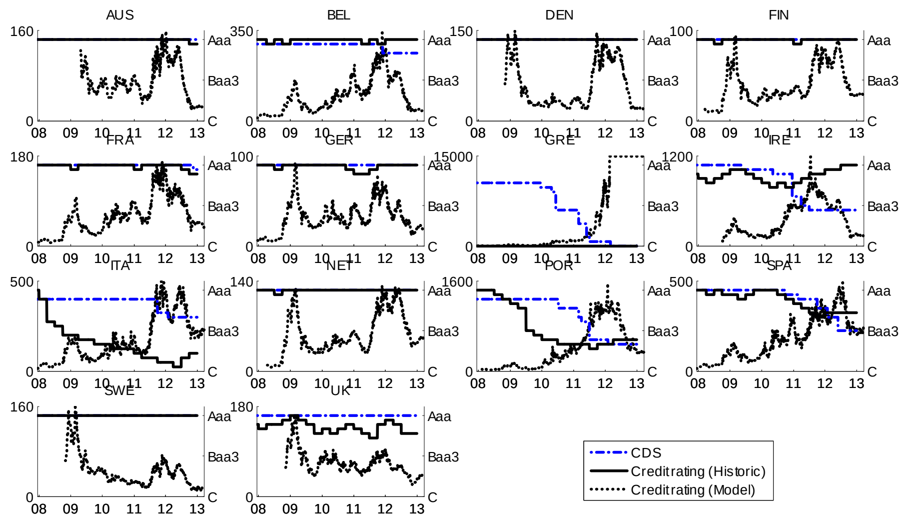
Notes: IG=Investment grade. Source: Authors' calculations.

credit ratings of EU14 countries began to occur from 2010 onwards. Using model-based credit ratings we observe that changes in the distribution begin in 2007-2008, reflecting the fact that the model-based ratings anticipate the 2010-2011 downgrades by the CRAs of several of the EU14 countries. By 2012 the distribution appears to be less skewed around the triple-A mark relative to the pre-crisis period. Table 9 in appendix D shows that these results for SF1-SF3 are not affected by the choice of debt limit. In addition we noted previously that several countries in the sample are downgraded from 2008 even under the MDL limit, which suggests that a shift in the cross-section distribution is observed even under the highest possible assessment of the borrowing capacity.

SF4, which relates to the variability over time of country credit ratings still holds: the composition of the four groups of countries has however changed. This emerges from Figure 9. Under the model-based measure of sovereign credit ratings only Denmark has a triple-A rating for the 1995-2012 period. Austria, Belgium, Finland, France, Germany, the Netherlands and Sweden have a rating ranging between triple-A and Aa. Ireland and the U.K. have a rating within the investment grade category; while Greece, Italy, Portugal and Spain are rated below investment grade at some stage over the period 1995-2012. The standard deviations within these four groups are 0, 0.26, 1.40, and 3.61,respectively.

SF5, on the relation between sovereign credit rating and the market perception of sovereign risk, is revisited in Figure 10 which shows the behavior of the historic and the model-based credit ratings during 2008-2012, together with the 5-year sovereign CDS prices. Several features emerge. First, the model-based credit ratings appear to display temporary downgrades in anticipation of subsequent temporary increases in CDS prices. This is clearly visible for Belgium (late 2009 and 2011), Finland (late 2008 and 2010), France (late 2008 and 2010), Germany (late 2008 and mid 2011), Ireland (mid 2010) and the Netherland (mid 2008). Second, the model-based credit ratings predict persistent downgrades in anticipation of a prolonged increase in CDS prices. This is clearly the case for Italy, Portugal and Spain, but not Greece only because the model-based rating

Figure 10: Sovereign credit ratings (historic and model-based) and 5-year credit default swap prices of EU14 countries, 14/12/2007 - 22/03/2013.



predicts a Greek default well before 2008. There are, however, still instances in which there is no clear relation between the credit ratings and CDS prices. For example, Denmark retains a model-based triple-A throughout the 2008-2012 period. Also the model-based ratings for the U.K. appear to be unrelated to movements in their CDS prices. The U.K. credit rating is downgraded from early 2008 coinciding with the sharp deterioration in U.K. public finances in the aftermath of the run on Northern Rock.

6 Conclusions

We have proposed a model-based analysis of sovereign credit ratings that reflects the ability of a country to use tax policy to repay its outstanding financial liabilities. The credit rating is obtained from the probability of sovereign default. Sovereign default probabilities at different time horizons are derived from an adaptation to the government sector of the formula for the probability of exercising an European call option in Black and Scholes (1976). The approach is, however, very general and can be extended to both private and public entities.

The empirical implementation in this paper involves four steps. First, it requires a prediction of the debt-GDP ratio, and a measure of the uncertainty surrounding this prediction, over a future horizon. We use a reduced-form macroeconomic model with time-varying parameters and time-varying volatility for these debt-GDP forecasts. Second, it needs an estimate of the maximum borrowing capacity, or debt limit, of the government. This may be derived from any source, including using ad hoc values. We estimate the debt limit using a standard open-economy DSGE macroeconomic model with distortionary taxation. Third, using the estimated distribution of the forecast of the debt-GDP ratio, we calculate the probability that, over a given horizon, it will exceed the estimated debt limit. Finally, we map this probability into a letter-grade credit rating using information on the observed default history of rated sovereign securities. We refer to this measure as a model-based credit rating because it involves models both for forecasting the debt-GDP ratio and for estimating the debt limit.

This model-based measure has two advantages. It provides investors with a transparent benchmark measure of the sovereign credit ratings as it is based on a narrow, but clear and specific, definition of the likelihood of default, namely, the ability of a country to repay its debt using financial savings generated by changes in fiscal policy. Second, by comparing differences between this model-based measure and the sovereign credit rating issued by the CRAs it is possible to determine the extent to which factors beyond fiscal policy may have contributed to the CRAs ratings.

These advantages are illustrated in our empirical analysis of the credit ratings of fourteen European countries from 1995 to 2012 where our main finding is that the model-based ratings downgrade of a number of European countries from 2008 whereas the CRAs start to downgrade them from 2010. The reason for the model-based findings is that, from mid-2007, due to large increases in expenditures and falls in tax revenues, the debt-GDP and deficit-GDP ratios show a significant deterioration in the fiscal stances of European countries. The consequence is increases in debt-GDP forecasts and falls in the estimated debt limits which increase the default probabilities and the likelihood of downgrades. This results in the cross-section distribution of EU credit ratings shifting away from triple-A and becoming more dispersed. Before 2007 the distribution was highly concentrated about triple-A. The historic ratings do not show this shift until 2010. This suggests that a model-based analysis of sovereign credit ratings would have picked up signals of an impending European debt crisis two years before the CRAs. An alternative interpretation is that the more positive judgment of the CRAs in 2008 and 2009 may have been due to taking account of additional factors to those that determine the fiscal stance and whether these would permit debt to be repaid. For countries with an independent domestic central bank, the most likely additional factor is the ability to repay debt using domestic monetary policy; for countries that have adopted the Euro the most likely additional factor may be confidence that the ECB would be willing to act as a lender of last resort and so help an indebted country to avoid leaving the common currency. Further analysis of these, or other, possible explanations for the difference between the two credit assessments is beyond the scope of this paper.

More generally, we do not make any judgemental comment about the ratings of the CRAs where they differ from the model-based ratings, as the official ratings may reflect the use of additional non-financial information when forming their credit ratings, and not just information about public finances. This is particularly true when downgrades in the credit ratings are also affected by private-sector finances, - in particular, bank-financed real estate loans - and not just public-sector finances. The model-based credit rating can only pick these up to the extent that public finance support for the banks is shown in the government budget constraint. In this way financial distress in the private sector rapidly transmitted itself to public finances, and hence to the country's model-based sovereign credit rating.

A number of possible extensions of this research are promising. For example, our computation of debt limits omits any consideration of the ability and willingness of policy makers to implement required fiscal changes. More appropriate debt limits might perhaps be obtained by incorporating political economy structures into DSGE models. A possibly even more promising refinement of the calculation of the debt limit may be obtained by allowing policy changes to government expenditures. Like Trabandt and Uhlig (2011), we find that most European countries are operating close to the peak of the Laffer hill for taxes. The IGBCL and the FL already incorporate anticipated changes in government expenditures. Nonetheless, an effective way to achieve fiscal consolidation might be through a discretionary unanticipated reduction in expenditures than by an increase in taxes. This might require a reformulation of the production function in the DSGE macroeconomic model, for example, by including both physical and human capital, with the latter being financed in full or in part from public expenditures.

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A Data

A.1 ROVAR

The variables used in the ROVAR are derived as follows. $\frac{b_t}{y_t}$ is constructed using annual series for gross debt-GDP ratio from Polito and Wickens (2011a) for the period 1970-1997; data for Portugal start from 1977. Data from 1998 to 2013 are taken from the OECD Economic Outlook No.92. The deficit-GDP ratio is constructed starting from annual data on total disbursements and total revenue of the general government as a proportion to GDP from the OECD Economic Outlook (Datastream, October 2012; mnemonics are XXOCFGU% for expenditure and XXOCFYRQ for revenue, with XX denoting the country acronym). The data range from 1970 to 2012, other than Portugal, for which revenue data are available from 1977. The missing observation for expenditure and revenue of Denmark in 1970 is taken from Polito and Wickens (2011a). The annual data for 2013 are taken from the OECD Economic Outlook No.92. $\frac{D_t}{y_t}$ is calculated as the difference between revenue and expenditure. Data for γ_t are quarterly observations on real GDP from the OECD Economic Outlook (Datastream, October 2012; XXOCFGVOD). Observations are available from 1970:1 to 2012:4. γ_t is computed as $400 \times \Delta \ln GDP$. π_t is constructed starting from quarterly data on the deflator from the OECD Economic Outlook (Datastream, October 2012; XXOCFGVOD). Observations are available from 1970:1 to 2012:4. π_t is measured as 400 times the logarithm of the deflator. r_t^s is derived from quarterly data on the short-term interest rate from the OECD Economic Outlook (Datastream, October 2012; XXOCFISTR). Data are available until 2012:4; but start from 1979:2 for Denmark, 1984:1 for Ireland, 1977:1 for Spain, 1982:1 for Sweden and 1971:1 for Italy. r_t^l is based on quarterly data on the long-term interest rate from the OECD Economic Outlook (Datastream, October 2012; XXOCFILTR). Data are available until 2012:4. Data start from 1992:4 for Greece and 1971:1 for Ireland. The missing initial observations for both r_t^s and r_t^l are derived by interpolating quarterly the corresponding annual observations from Polito and Wickens (2011a). e_t is derived starting from annual data on the nominal effective exchange rate from the OECD Economic Outlook (Datastream, October 2012; XXOCFEXE). These data range from 1970 to 2012. The annual data for 2013 are taken from the OECD Economic Outlook No.91. The implied depreciation rate included is computed as 400 times the first difference of the log of the data. $\frac{x_t}{y_t}$ is derived starting from annual data are available from the OECD Economic Outlook (Datastream, October 2012; XXOCFC%G). Data are available from 1988 for Denmark and from 1975 for all other countries. Data for Denmark from 1975 to 1988 are from the World Bank WDI (Datastream, October 2012; DKWDLTLJR). The annual data for 2013 are taken from the OECD Economic Outlook No.91. π_t^o refers to the crude oil price, spot Brent, from the OECD Economic Outlook (Datastream, October 2012, OCOCBRNTB). This is available from 1960:1 to 2012:4. Datastream report this as an AR series. We interpret this as meaning that the data are already annualized. Oil inflation is calculated by multiplying by 100 the first difference of the log of the data. Quarterly values of $\frac{b_t}{y_t}$, $\frac{D_t}{y_t}$, e_t and $\frac{x_t}{y_t}$, are determined using linear interpolation on the corresponding annual data. It is assumed that annual observations correspond values in the second quarter. Thus the quarterly observations of these variables range from 1975:2 to 2013:2.

A.2 Government accounts

We have taken from Datastream (October 2012) the following OECD Economic Outlook government account data: total government receipts (% GDP, XXOCFYRQ), Taxes on production and imports (Millions, XXOCFITX), Total direct taxes (Millions, XXOCFTAX), Social security contributions received (Millions, XXOCFSSC), Gross government interest receipts (Millions, XXOCFIRC), Gross government interest paid (Millions, XXOCFIPY), Social security contributions paid (Millions, XXOCFSSB), Capital transfer paid (Millions, XXOCFCTT), Total disbursements (% GDP, XXOCFGU%) and nominal GDP (Millions, XXOCFGPN).

Data are annual and available for 1977-2012 for Portugal, 1971-2012 for Denmark and 1970-2012 for all other countries. The missing observation for Denmark in 1970 is replaced using the 1971-1973 average value. Where required all data are scaled by nominal GDP. g_t/y_t is calculated by subtracting social security, capital transfers and gross interest rates paid by the government from total disbursements. v_t/y_t is calculated by adding direct taxes, taxes on production and social security received by the government. z_t/y_t is computed by subtracting non-tax revenue from social security and capital transfers paid by the government. Non-tax revenue is calculated by subtracting v_t/y_t and interest revenue from total revenue.

A.3 Average tax rates

Annual data from 1995 to 2010 on implicit tax rates (ITRs) on capital, labor and consumption are available from Eurostat (2012). The dataset also provides data on total tax revenue, and tax revenue from capital, consumption and labor in each year from 1995 to 2010. A number of observations are missing in some countries. To retrieve these, we have first calculated the ratios of each ITR and the revenue it generates. These ratios are fairly stable over time. The missing IRTs are then determined by multiplying these ratios (either the average or the initial or the value in the final year depending on the missing ITR) by the tax revenue generated in each year.

We then employ data on tax revenue from the OECD Economic Outlook described in Appendix A.2 to infer ITRs for 2011 and 2012.

This is done as follows. First, we add revenue from direct taxes, production and imports and social security contributions from the OECD Economic Outlook. Second, we compute the ratio of revenue from consumption labor and capital in terms of the total tax revenue using the EUROSTAT data. This gives the shares of consumption, labor and capital revenue as a proportion of the total tax revenue from 1995 to 2010. Third, we compute the difference between the average total tax revenues from EUROSTAT and the OECD Economic Outlook from 1995 to 2010 (the OECD tax revenue is higher than that from EUROSTAT in each year). This defines the adjustment required to reconcile the two tax revenues. Fourth, we multiply the share of consumption, labor and capital in 2010 by the total tax revenue from the OECD (minus the adjustment) in

2011 and 2012. This gives the value of revenue from consumption, labor and capital as a proportion to GDP in those years which can be used to retrieve the corresponding ITRs. Finally, we use linear interpolation on the annual data to derive quarterly series of the three ITRs. This gives 69 observations, from 1995:4 to 2012:4.

B Stationary equilibrium debt-GDP ratio

The first-order conditions for the consumption of domestic and foreign goods, labor, capital, domestic and net foreign assets that are derived from the household maximization problem are:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c_t^H} &= \beta^t u_{c,t} p_t^H - \lambda_t (1+\tau_t^c) p_t^H = 0, \\
\frac{\partial \mathcal{L}}{\partial c_t^F} &= \beta^t u_{c,t} p_t^F - \lambda_t (1+\tau_t^c) p_t^F = 0, \\
\frac{\partial \mathcal{L}}{\partial n_t} &= \beta^t u_{n,t} + \lambda_t (1-\tau_t^n) w_t = 0, \\
\frac{\partial \mathcal{L}}{\partial k_t} &= E_t \left\{ \lambda_{t+1} \left[1 + \left(r_{t+1}^k - \delta \right) \left(1 - \tau_{t+1}^k \right) \right] \right\} - \lambda_t = 0 \\
\frac{\partial \mathcal{L}}{\partial b_t^D} &= E_t \left\{ \lambda_{t+1} \left[\left(1 - \Xi_{t+1}^D \right) + \left(1 - \xi_{t+1}^D \right) r_{t+1} \right] \right\} - \lambda_t = 0 \\
\frac{\partial \mathcal{L}}{\partial f_t} &= E_t \left[\lambda_{t+1} s_{t+1} \left(1 + r_{t+1}^* \right) \right] - s_t \lambda_t = 0.
\end{aligned}$$

Given (12) and (8), the Euler equations for the intratemporal equilibrium between labor and consumption, the income identity and the no-arbitrage equilibrium conditions are:

$$\begin{aligned}
\frac{E_t \left[(1+\tau_{t+1}^c) c_{t+1} \right]}{(1+\tau_t^c) c_t} &= \beta \left\{ 1 + E_t \left\{ \left[\alpha k_{t+1}^{\alpha-1} \left(A_{t+1} n_{t+1} \right)^{1-\alpha} - \delta \right] \left(1 - \tau_{t+1}^k \right) \right\} \right\} \\
\psi \frac{c_t}{1-n_t} &= \frac{(1-\tau_t^n)}{(1+\tau_t^c)} (1-\alpha) A_t k_t^\alpha \left(A_t n_t \right)^{-\alpha} \\
k_t^\alpha \left(A_t n_t \right)^{1-\alpha} &= c_t + g_t + k_t - (1-\delta) k_{t-1} + x_t \\
1 + E_t \left[\left(r_{t+1}^k - \delta \right) \left(1 - \tau_{t+1}^k \right) \right] &= E_t \left[\left(1 - \Xi_{t+1}^D \right) + \left(1 - \xi_{t+1}^D \right) r_{t+1} \right] = E_t \left[\frac{s_{t+1}}{s_t} \left(1 + r_{t+1}^* \right) \right].
\end{aligned}$$

The stationary equilibrium solution for capital is in the main text, while those for consumption and labor are $c = \Omega k - g - x$ and $n = \varphi k$, respectively, with Ω and φ as defined in the main text. The stationary equilibrium solutions for output, wages, and net trade are: $y = k^\alpha (An)^{1-\alpha}$, $r^k = \alpha k^{\alpha-1} (An)^{1-\alpha}$, $w = A(1-\alpha) k^\alpha (An)^{-\alpha}$ and $x = r^* \left(b^F - sf \right)$ respectively. The stationary equilibrium solution for gross rates of returns is

$$1 + r^* = \left(1 - \Xi^D \right) + \left(1 - \xi^D \right) r = 1 + \left[\alpha k^{\alpha-1} (An)^{1-\alpha} - \delta \right] \left(1 - \tau^k \right).$$

which gives the stationary-equilibrium rate of interest on domestic bonds in equation (17).

These can be combined to obtain stationary equilibrium values for the capital-output ratio, $\frac{k}{y} = \left[\frac{\beta^{-1}-1}{\alpha(1-\tau^k)} + \frac{\delta}{\alpha}\right]^{-1}$, the output-labor ratio, $\frac{y}{n} = A\left[\frac{\beta^{-1}-1}{\alpha(1-\tau^k)} + \frac{\delta}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}$, the consumption-output ratio $\frac{c}{y} = \chi\left(\frac{1}{\varphi k} - 1\right)$ and the real wage, $w = (1-\alpha)A\left[\frac{\beta^{-1}-1}{\alpha(1-\tau^k)} + \frac{\delta}{\alpha}\right]^{-\frac{\alpha}{1-\alpha}}$; with χ as defined in the main text. Finally, the stationary equilibrium debt-GDP ratio is derived from the equilibrium solution to the GBC

$$\frac{b}{y} = \frac{1}{r}\left(\frac{v}{y} - \frac{g}{y} - \frac{z}{y}\right),$$

where $\frac{v}{y} = \tau^c\frac{c}{y} + \tau^n w\frac{n}{y} + \tau^k\left(r^k - \delta\right)\frac{k}{y} + \frac{q}{y}$, $b = b^F = b^D$ and r is defined in equation (17). The tax-GDP ratio can therefore be formulated as

$$\frac{v}{y} = \tau^c\chi\left(\frac{1}{\varphi k} - 1\right) + \tau^n(1-\alpha) + \tau^k\left\{\alpha - \delta\left[\frac{\beta^{-1}-1}{\alpha(1-\tau^k)} + \frac{\delta}{\alpha}\right]^{-1}\right\} + \frac{q}{y}.$$

From this we obtain the stationary equilibrium debt-GDP ratio in equation (13).

C Solution algorithm

The Markov Chain Monte Carlo simulation involves the following steps. *Step 1: Estimate the time-varying volatility of technology shocks.* We use the log transformation of equation (8) to derive a time-series for the logarithm of technological progress ($\ln A_t = \frac{1}{1-\alpha}\left[\ln y_t - \alpha\ln k_t - (1-\alpha)\ln n_t\right]$) over the period 1970:1-2012:2. This uses data on total employment, gross fixed capital formation and real GDP. Data on total employment (Datastream, Thousands Persons, XXOCFEMPO) are quarterly for all countries other than Greece and start before 1970 (we use data from West Germany prior 1991). Data for Greece, annual from 1961 to 2012, are interpolated to retrieve the corresponding quarterly series. Data on Gross Fixed Capital formation (Datastream, Millions Euro, 2005 prices, XXOCFINVD) are quarterly for all countries. Data for Italy are based on current prices; the constant-price series are determined using the corresponding deflator. For Greece, data are available on an annual basis, so quarterly series is determined through linear interpolation. Real GDP data are described in Appendix A.1. We assume a capital share of output of 0.3. We then measure the rolling-window (40 quarters) standard deviation of the derived series for $\ln A_t$ which is used as proxy for the time-varying volatility of technological progress. We employ data for the period 1995:4 to 2012:4. *Step 2: Estimate the time-varying mean and volatility of $\frac{g_t}{y_t}$ and $\frac{z_t}{y_t}$.* These are derived by calculating rolling-window (of 40 periods) means and standard deviations for the time series of government expenditure-GDP and transfers-GDP described in Appendix A.2. We employ data for the period 1995:4 to 2012:4. *Step 3: Estimate Laffer hills.* For each country we simulate numerically the stationary equilibrium solution of the model over the period 1995:4-2012:4 using rolling-window mean values of $\frac{g_t}{y_t}$ (see step 2) and the tax rates on consumption (see A.3). Each quarter we allow τ^n and τ^k to range from 0.01 to 0.99 (with increase of 0.01). We then use grid search to find the combination of τ^n and τ^k that maximizes the revenue-GDP ratio in each quarter. This yields the series $\tau^{n,\max}$ and $\tau^{k,\max}$ that correspond to the peak of the Laffer hill at each quarter of the sample period. The simulation is carried out using β=0.95, δ=0.012, ψ=0.6, and $\overline{A} = 1$. The exchange rate is normalized, so that $s_t = 1$. *Step 4: Stochastic simulation of the shocks.* We assume that the natural logarithms of $\frac{g_t}{y_t}$, $\frac{z_t}{y_t}$ and A_t follow an AR(1) process with time-varying volatility (see steps 1 and 2) and that $\frac{g_t}{y_t}$ and $\frac{z_t}{y_t}$ have time-varying means, (see step 2). The mean of the technological progress is normalized to 1. Thus we specify $\ln h_t = \left(1 - \rho^h\right) \ln \overline{h}_t + \rho^h \ln h_{t-1} + \epsilon_t^h$, where $\epsilon_t^h \sim N\left(0, \sigma_h^2\right)$ and $h = \{\frac{g_t}{y_t}, \frac{z_t}{y_t}, A_t\}$. We simulate these AR(1) process 200 times each quarter over the period 1995:4-2012:4 using a constant mean reversion coefficient $\rho^h = 0.553$. *Step 5: Compute time-varying stationary equilibrium.* Using the tax rates from either Appendix 3 or step 3, we calculate the steady-state solution of the model and the implied consumption path, for each of the 200 values of $\frac{g_t}{y_t}$ and A_t simulated from 4. *Step 6: Compute time-varying debt-GDP limits.* Using the simulated values of $\frac{v_t^{\max}}{y_t}$, $\frac{v_t}{y_t}$, $\frac{g_t}{y_t}$ and $\frac{z_t}{y_t}$ we calculate the debt limits in equations (13)-(16). We employ country-specific discount rates using the sample average of the long-run interest rate r_t^l. The implied annual discount factors are: 0.957 (AUS), 0.956 (BEL), 0.956 (DEN), 0.956 (FIN), 0.957 (FRA), 0.959 (GER), 0.923 (GRE), 0.948 (IRE), 0.948 (ITA), 0.958 (NET), 0.943 (POR), 0.950 (SPA), 0.954 (SWE) and 0.952 (UK). *Step 7: Compute posterior distribution of the debt-GDP limits.* We repeat steps 4-6 10000 times to obtain the posterior means and standard deviations of each of the four debt limits.

D Further results

Table 8: Model-based sovereign credit rating of EU14 countries, 1995:4-2012:4.

Time	Rating	Time	Rating	Time	Rating	Time	Rating	Time	Rating
AUS		GER		ITA		SPA		UK	
Q4 1995	Aaa	Q4 1995	Aaa	Q4 1995	SG	Q4 1995	SG	Q4 1995	Aaa
Q4 2012	Aa1	Q3 2008	Aa1	Q4 1999	Baa3	Q1 1999	Baa3	Q3 2007	Aa1
BEL		Q4 2008	Aaa	Q1 2000	Ba1	Q2 1999	Baa2	Q4 2007	Aa2
Q4 1995	Aaa	Q4 2010	Aa1	Q2 2000	Baa3	Q3 1999	Baa1	Q1 2008	Aa3
Q4 2004	Aa1	Q1 2011	Aa2	Q4 2000	Baa2	Q4 1999	A3	Q2 2008	Aa2
Q1 2005	Aa2	Q3 2011	Aa1	Q1 2001	Baa3	Q1 2000	A2	Q4 2008	Aa1
Q2 2005	Aa1	Q4 2011	Aaa	Q3 2001	Baa2	Q2 2000	A1	Q1 2009	Aaa
Q3 2005	Aaa	GRE		Q4 2001	Baa3	Q3 2000	Aa3	Q2 2009	Aa1
Q2 2008	Aa1	Q4 1995	C	Q2 2002	Baa2	Q4 2000	Aa2	Q3 2009	Aa2
Q3 2008	Aaa	IRE		Q3 2002	Baa1	Q1 2001	Aa1	Q4 2009	A1
Q4 2008	Aa1	Q4 1995	Aaa	Q4 2003	Baa2	Q2 2001	Aaa	Q1 2010	Aa3
Q1 2009	Aaa	Q3 2007	Aa1	Q3 2004	Baa1	Q3 2007	Aa1	Q2 2010	A1
Q2 2011	Aa1	Q4 2007	Aa2	Q4 2004	A2	Q4 2007	Aaa	Q3 2010	Aa3
Q3 2011	Aaa	Q1 2008	Aa3	Q1 2005	Baa1	Q2 2008	Aa1	Q4 2010	Aa2
Q4 2011	Aa1	Q2 2008	A1	Q2 2005	Baa3	Q3 2008	Aaa	Q1 2011	Aa3
Q1 2012	Aaa	Q3 2008	Aa3	Q4 2006	Baa2	Q4 2008	Aa1	Q2 2011	A1
DEN		Q4 2008	Aa2	Q1 2007	Baa1	Q2 2009	Aa2	Q3 2011	A2
Q4 1995	Aaa	Q1 2009	Aa1	Q2 2007	A1	Q3 2009	Aa1	Q4 2011	Aa2
FIN		Q3 2009	Aa2	Q3 2007	Aa2	Q4 2009	Aaa	Q1 2012	Aa1
Q4 1995	Aa1	Q4 2009	Aa3	Q4 2007	Aaa	Q3 2010	Aa1	Q2 2012	Aa2
Q1 1996	Aaa	Q1 2010	A1	Q1 2008	Aa2	Q4 2010	Aa2	Q3 2012	A1
Q3 2008	Aa1	Q2 2010	A2	Q2 2008	Baa1	Q1 2011	Aa3		
Q4 2008	Aaa	Q3 2010	A1	Q3 2008	Baa2	Q2 2011	A1		
Q1 2011	Aa1	Q4 2010	A2	Q4 2008	SG	Q3 2011	A2		
Q2 2011	Aaa	Q1 2011	A1	NET		Q4 2011	A3		
FRA		Q2 2011	Aa3	Q4 1995	Aaa	Q1 2012	A2		
Q4 1995	Aaa	Q3 2011	Aa2	Q3 2008	Aa1	SWE			
Q3 1996	Aa1	Q1 2012	Aa1	Q4 2008	Aaa	Q4 1995	Aaa		
Q4 1996	Aa2	Q3 2012	Aaa	POR		Q4 2002	Aa1		
Q2 1997	Aa1			Q4 1995	Aaa	Q1 2003	Aaa		
Q4 1997	Aaa			Q3 2002	Aa1	Q2 2003	Aa1		
Q1 2009	Aa1			Q4 2002	Aa2	Q3 2003	Aaa		
Q2 2009	Aaa			Q1 2003	Aa1				
Q1 2011	Aa1			Q3 2003	Aaa				
Q2 2011	Aaa			Q3 2008	Aa1				
Q3 2012	Aa1			Q4 2008	Aa2				
Q4 2012	Aa2			Q1 2009	Aa3				
				Q2 2009	A1				
				Q3 2009	Baa3				
				Q4 2009	SG				

Note: SG = Speculative grading. Source: Authors' calculations based on data in Figure 12.

Table 9: Distribution of the model-based sovereign credit rating of EU14 countries at selected dates based on the IGBCL and FL limits.

	1995	2000	2005	2006	2007	2008	2009	2010	2011	2012
					Debt limit: IGBCL					
Aaa	71%	71%	86%	86%	71%	43%	57%	50%	50%	43%
Aa	7%	7%	0%	0%	14%	36%	14%	14%	14%	21%
A	0%	7%	0%	0%	0%	7%	7%	7%	7%	0%
Baa	0%	0%	0%	0%	0%	0%	7%	7%	7%	0%
Ba	0%	0%	0%	0%	0%	0%	0%	7%	0%	14%
B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Caa-C	21%	14%	14%	14%	14%	14%	14%	14%	21%	21%
IG	79%	86%	86%	86%	86%	86%	86%	79%	79%	64%
					Debt limit: FL					
Aaa	79%	86%	93%	93%	79%	57%	64%	50%	57%	50%
Aa	14%	7%	0%	0%	14%	36%	21%	29%	14%	36%
A	0%	0%	0%	0%	0%	0%	7%	14%	14%	0%
Baa	0%	7%	0%	0%	0%	0%	0%	0%	7%	7%
Ba	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
B	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%
Caa-C	7%	0%	7%	7%	7%	7%	7%	7%	7%	7%
IG	93%	100%	93%	93%	93%	93%	93%	93%	93%	93%

Notes: IG=Investment grade. Source: Authors' calculations.