Sovereign Debt Sustainability in Advanced Economies

Fabrice Collard (U. of Bern)
Michel Habib (U. of Zurich, Swiss Finance Institute, and CEPR)
Jean-Charles Rochet
(U. of Zurich, Swiss Finance Institute, and Toulouse School of Economics)

May 2014

Outline

- Motivation and Central Insight
- An Example: Serbia
- Literature Review
- Model
- Calibration and Results
- Variation over Time
- Extensions
- Conclusion

Motivation and Central Insight

There has been a tremendous increase in advanced economies' debt-to-GDP ratios, to levels perhaps previously thought impossible.

This begs the question of how high a country can (more or less safely) go.

In other words, what is a country's *maximum sustainable debt* (MSD)?

A country can borrow only as much as lenders are willing to provide.

Lenders base their decision on:

- the mean and the volatility of the country's growth rate in GDP,
- the primary surpluses the country can achieve,
- the country's attitude to debt repayment, and
- the expectation of the amount of debt that can be raised in the future to service the debt raised now.

There is a ‘borrowing multiplier’ that raises a country’s borrowing well above what it would be absent the ability to service maturing debt out of new debt’s proceeds.

We seek the conditions under which the multiplier is finite.

Equivalently, we seek a fixed point in borrowing. Any level of debt above that point would have to rely on future borrowing constituting an ever larger fraction of GDP, i.e., a bubble.

That fixed point defines a country’s *maximum sustainable borrowing* (MSB); the corresponding amount to be repaid constitutes the country’s MSD.

Perhaps our most interesting result is that there is a stark asymmetry in the relation between a country's sovereign debt and the probability of defaulting on that debt on either side of MSD.

PD increases in debt quite slowly below MSD, fairly rapidly above.

This is because MSD equates the country's marginal and average interest rates, somewhat similarly to the manner in which a firm's cost minimizing output equates the firm's marginal and average costs.

An Example: Serbia

Over the period 2000-2012, Serbia had mean annual growth 2.96% and annual growth volatility 6.35%.

Its MSD at 5% maximum primary surplus is 75.02% with associated probability of default 2.96%.

In 2010, with actual debt 44.5%, its actual PD was near 0%.

Literature Review

Our attempt at computing *maximum* debt follows in the footsteps of Bohn (1998, 2008), Ghosh, Kim, Mendoza, Ostry, and Qureshi (2011), and Tanner (2013).

Our focus on maximum debt distinguishes our paper from the overwhelming majority of papers on sovereign debt which, in the spirit of Eaton and Gersovitz's (1981) seminal paper, have sought to estimate a country's *optimal* (as opposed to *maximal*) sovereign debt under the assumption of *strategic* (as opposed to *excusable*) default.

The notion of excusable default was introduced by Grossman and Van Huyck (1981): a government defaults only when the sum of government income and debt issuance proceeds falls short of debt service requirements.

Levy-Yeyati and Panizza (2011) provide strong evidence of governments' reluctance to default.

Tomz (2007) reads the historical evidence as supporting the view that it is only governments that engage in strategic default that are punished by way of high future costs of borrowing.

Borensztein and Panizza (2008) and Malone (2011) find that governments that default see a marked decline in their prospects for reelection. Broner, Martin, and Ventura (2010) explain why this may be so even when much of the debt is initially held by foreigners. Gümbel and Sussman (2009) argue that the median voter may favor debt repayment even when much of the benefit accrues to foreign bondholders.

Bolton and Jeanne (2011) argue that government default may jeopardize the functioning of the banking system, because government bonds provide the collateral for interbank loans.

Model

Denote

y_t: GDP in period t

α: maximum primary surplus (MPS) as fraction of GDP

b_t: proceeds from debt issuance in period t as fraction of GDP y_t

d_t: face value of debt issued in period t and due in period $t+1$, as fraction of GDP y_t

$g_{t+1} = y_{t+1}/y_t$: rate of growth in GDP, distributed i.i.d. with cdf $F(.)$ and pdf $f(.)$

r: risk-free interest rate

Maximum borrowing proceeds $b_t y_t$ for a given face value $d_t y_t$ and future borrowing proceeds $b_{t+1} y_{t+1}$ are

$$b_t y_t = \frac{\Pr[(\alpha + b_{t+1}) y_{t+1} > d_t y_t] d_t y_t}{1 + r}$$

The minimum growth rate necessary for default to be avoided, that is, for $(\alpha + b_{t+1}) y_{t+1} \geq d_t y_t$, is

$$x_t = \frac{d_t}{\alpha + b_{t+1}}$$

Rearranging, maximum borrowing proceeds b_t for given future borrowing proceeds b_{t+1} can be shown to equal

$$b_t = \max_{x_t} \frac{1}{1 + r} [1 - F(x_t)](\alpha + b_{t+1}) x_t$$

We define the borrowing factor γ to be

$$\gamma \equiv \max_{x}[1-F(x)]x < E[g] \equiv \bar{g}$$

and substitute into the equation for b_t and expand to obtain

$$b_t = \frac{\alpha\gamma}{1+r}\left[1+\frac{\gamma}{1+r}+\left(\frac{\gamma}{1+r}\right)^2+...\right] = b_S\Gamma$$

where $b_S \equiv \alpha\gamma/(1+r)$ defines maximum *static* borrowing, the maximum amount that can be borrowed when the country cannot rely on future borrowing to repay present borrowing and the borrowing multiplier Γ is

$$\Gamma \equiv 1+\frac{\gamma}{1+r}+\left(\frac{\gamma}{1+r}\right)^2+...$$

When $\gamma < 1+r$, the borrowing multiplier is finite and so is MSB b_M

$$b_M \equiv \frac{\alpha\gamma}{1+r-\gamma}$$

It is interesting to compare MSB with what one might call *Maximum Sustainable Equity* (MSE), the proceeds the country would obtain it is could sell an equity claim to investors rather than a debt claim

$$b_E \equiv \frac{\alpha \bar{g}}{1 + r - \bar{g}} > b_M$$

From MSB b_M, we can determine MSD d_M

$$d_M \equiv (\alpha + b_M) x_M$$

where x_M is the minimum growth rate necessary to avoid default when the country owes d_M.

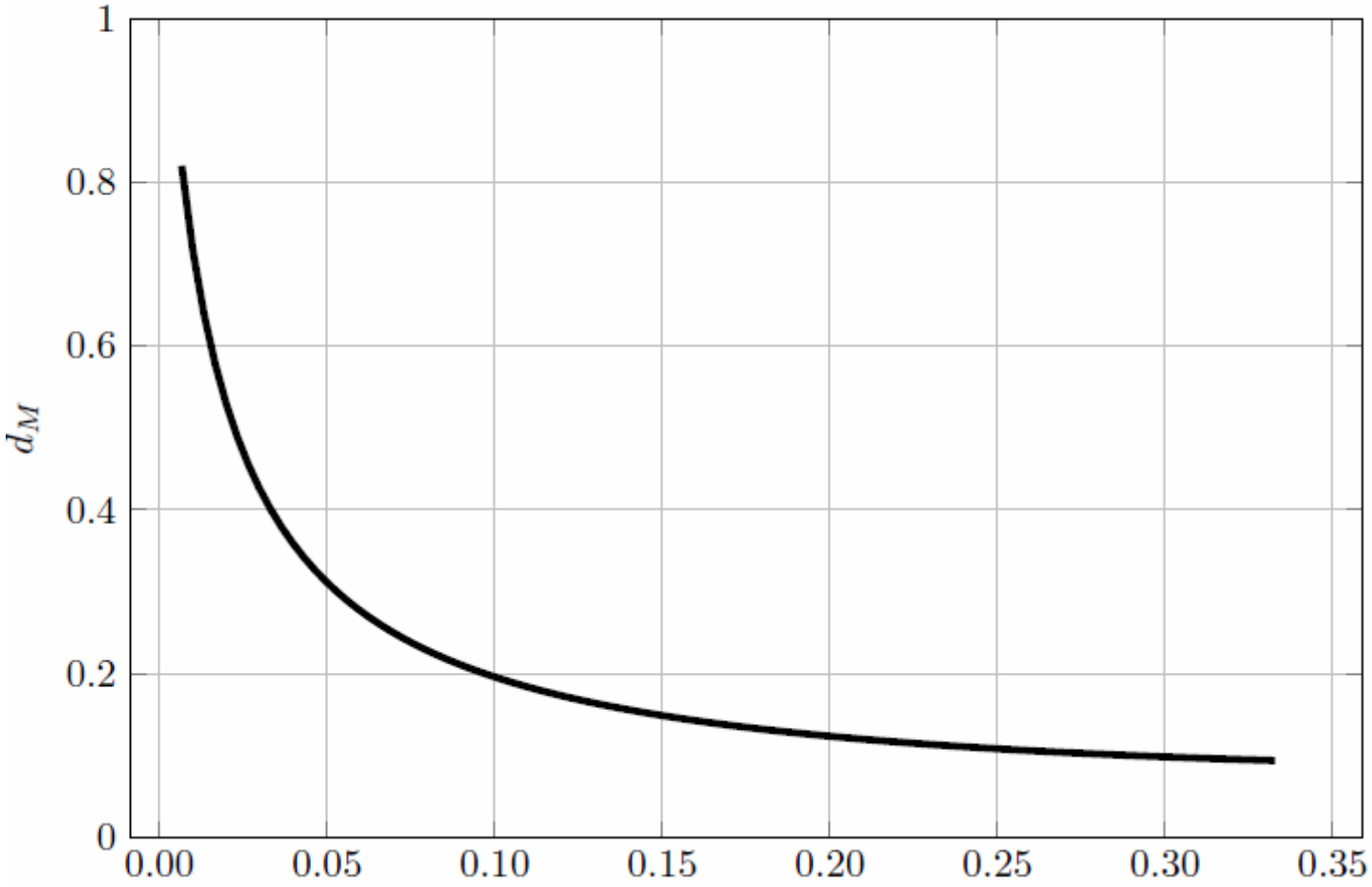
The probability of default is $PD(d_M) \equiv F(x_M) = F(d_M/(\alpha + b_M))$ when the country MSD owes d_M; it is $PD(d) \equiv F(d/(\alpha + b_M))$ when the country owes debt d.

That the country does its utmost to avoid default implies that it borrows the most in order to service its debt.

Under the assumption that $\ln(g) \sim N(\mu, \sigma^2)$, we can show that

- MSB is increasing in the mean growth rate and the MPS; it is decreasing in growth rate volatility for low PD and in the risk-free interest rate.
- MSD is increasing in the mean growth rate and the MPS and decreasing in the risk-free interest rate for low PD.
- PD is decreasing in the mean growth rate and the MPS; it is increasing in growth rate volatility for low PD and in the risk-free interest rate.

Figure 1: Maximum Sustainable Debt as a Function of Growth Rate Volatility



An important property of MSD is that it satisfies the relation

$$R'(d_M) = \frac{R(d_M)}{d_M}$$

In words, MSD d_M equates the marginal and average interest rates.

The average interest rate therefore attains its minimum at MSD. This implies that the interest rate increases rather slowly below MSD and rather rapidly above.

What is true of the interest is true of the probability of default: PD increases slowly below MSD and rapidly above.

Figure 4: Marginal and Average Interest Rates ($\alpha = 5\%$)

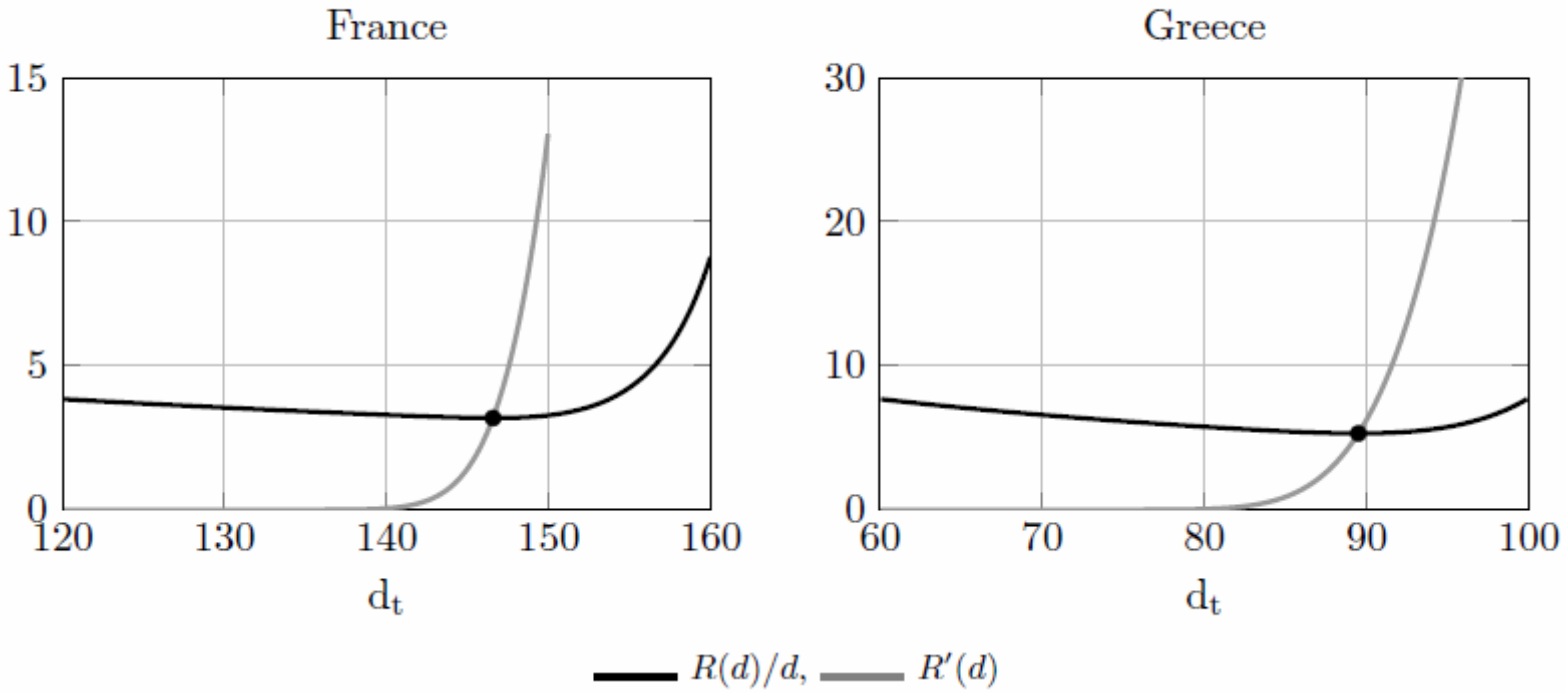
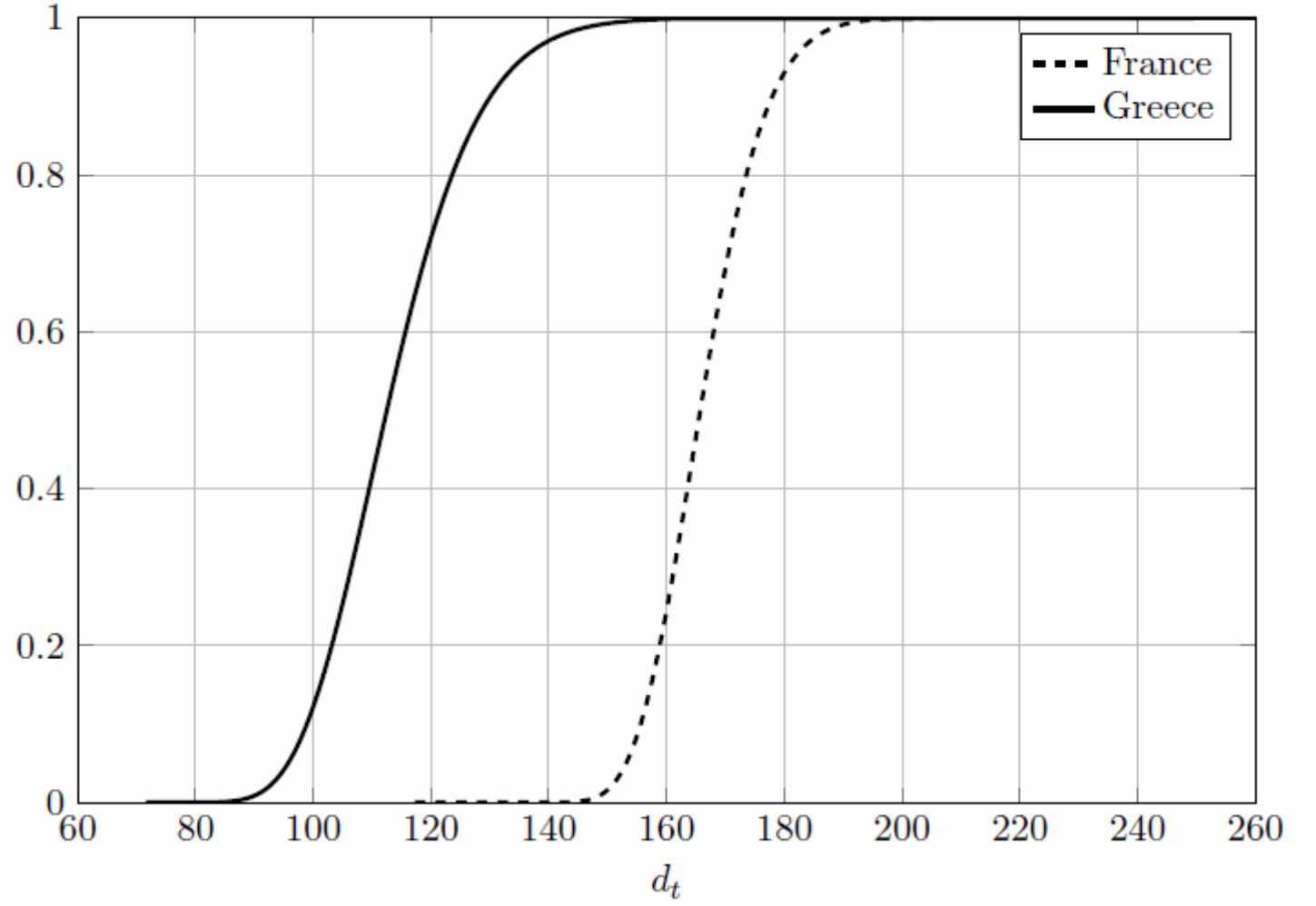
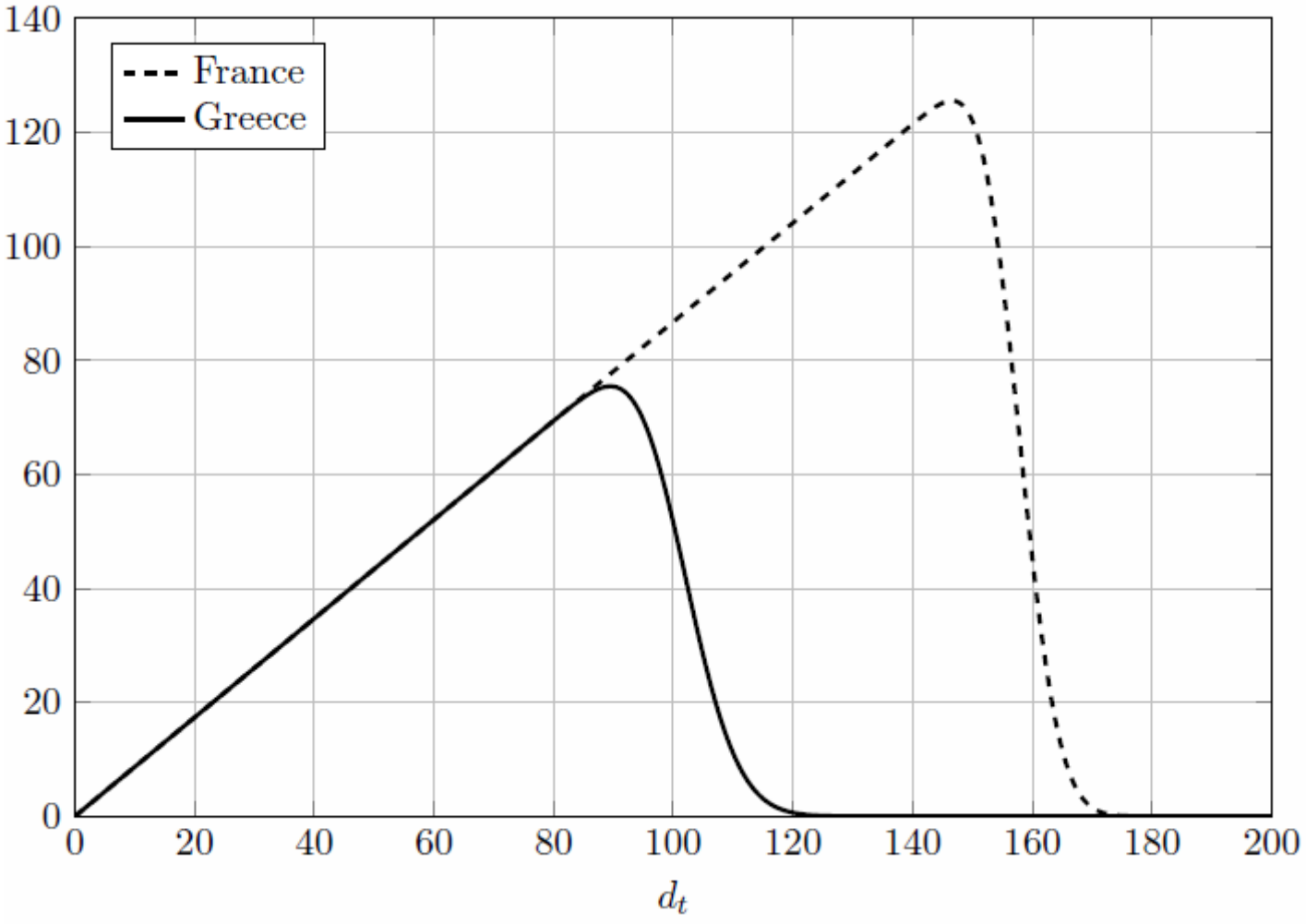


Figure 3: Default Probability ($\alpha = 5\%$)



The asymmetry in PD is reflected in the shape of the ‘Laffer Curve’ that relates borrowing proceeds to the face value of debt.

Figure 2: Borrowing proceeds as a function of nominal debt ($\alpha = 5\%$)



Data

Table 1: When it comes to debt, Australia is indeed the 'lucky country.'

Table 1: Data (%)

Country	μ	σ	d_{2010}	MPS
Australia	1.91	3.30	11.30	4.16
Austria	2.17	3.65	70.00	3.32
Belgium	1.88	3.71	100.20	6.84
Canada	1.72	4.10	81.70	10.05
Denmark	1.74	4.41	40.60	7.10
Finland	2.21	5.91	50.00	9.82
France	1.99	3.26	84.20	1.36
Germany	1.89	3.88	78.80	4.34
Greece	1.56	6.65	144.00	4.37
Hungary	1.98	5.12	72.60	8.39
Iceland	2.25	5.92	115.58	8.47
Ireland	3.30	5.52	93.60	6.74
Italy	1.67	4.53	117.50	6.51
Korea	5.75	7.39	32.00	6.44
Netherlands	1.84	3.41	67.40	5.62
New Zealand	1.12	4.53	31.00	7.73
Norway	2.37	2.84	54.30	20.25
Portugal	2.26	6.13	83.10	0.23
Spain	1.99	3.36	63.50	4.01
Sweden	1.71	4.40	41.70	7.05
Switzerland	1.01	4.10	39.50	3.05
United Kingdom	1.93	4.17	76.70	6.31
United States	1.75	4.16	92.70	5.09

Note: MPS denotes the historical maximum primary surplus, $\alpha = \max_t\{PS/Y\}$.

Table 2: Many governments clearly listened all too eagerly to economists' injunctions to stimulate their economies at the outset of the financial crisis.

Table 2: Debt-to-GDP Ratios (%)

	1980	min(D/Y)	E(D/Y)	max(D/Y)	2010
Australia	18.70	4.50	12.74	21.00	11.30
Austria	35.30	35.30	58.46	70.00	70.00
Belgium	65.80	65.80	107.71	134.20	100.20
Canada[(a)]	71.10	65.10	82.84	101.70	81.70
Denmark	35.00	27.90	57.06	79.80	40.60
Finland	11.00	11.00	34.32	57.70	50.00
France	20.70	20.70	48.65	84.20	84.20
Germany	30.00	30.00	52.05	78.80	78.80
Greece	24.60	24.60	86.57	144.00	144.00
Hungary[(b)]	156.20	55.70	91.69	172.00	72.60
Iceland	25.50	23.00	44.09	115.58	115.58
Ireland	65.20	24.80	68.86	109.20	93.60
Italy	53.50	53.50	97.77	120.10	117.50
Korea	14.10	8.90	20.70	36.80	32.00
Netherlands	45.40	45.40	63.13	77.20	67.40
New Zealand	45.00	17.40	42.82	71.60	31.00
Norway	47.30	28.90	42.51	60.50	54.30
Portugal	30.50	30.50	55.77	83.10	83.10
Spain	16.40	16.40	46.62	67.40	63.50
Sweden	39.30	37.60	54.67	73.20	41.70
Switzerland[(c)]	38.30	31.00	44.06	55.30	39.50
United Kingdom	46.10	31.30	44.65	76.70	76.70
United States	42.10	41.20	62.28	92.70	92.70

Note: (a) Column 1980 reports the number for 1988. (b) Column 1980 reports the numbe for 1991. (c) Column 1980 reports the number for 1983.

Calibration and Results

Table 3: High growth has its privileges, high MPS too.

Table 3: Maximum Sustainable Debt d_M (%)

Country	$\alpha = 5.00\%$ (1)	MRR (2)	TVR (3)	CATA (4)	$\alpha = 4.00\%$ (5)	MPS (6)
Australia	142.25	144.25	115.79	95.37	113.80	118.49
Austria	145.14	147.43	119.43	93.86	116.11	96.40
Belgium	132.73	134.87	113.19	87.81	106.18	181.50
Canada	120.75	122.97	108.33	79.80	96.60	242.70
Denmark	116.99	119.33	107.42	76.51	93.59	166.11
Finland	110.18	113.32	108.90	68.31	88.15	216.32
France	146.56	148.58	117.65	97.77	117.24	39.98
Germany	130.13	132.35	112.70	85.48	104.10	112.88
Greece	89.49	92.51	94.49	58.57	71.59	78.18
Hungary	113.92	116.65	108.47	72.19	91.14	191.25
Iceland	110.96	114.12	109.52	68.60	88.77	188.07
Ireland	153.02	156.89	134.74	85.64	122.42	206.33
Italy	113.23	115.58	105.52	74.20	90.58	147.53
Korea	281.74	291.34	205.10	116.77	225.39	362.77
Netherlands	137.37	139.38	113.97	92.15	109.90	154.53
New Zealand	100.48	102.60	96.66	68.12	80.38	155.37
Norway	177.91	179.97	128.33	116.69	142.33	720.37
Portugal	108.78	112.02	108.66	67.30	87.02	5.10
Spain	144.16	146.22	117.10	95.79	115.33	115.57
Sweden	116.14	118.46	106.85	76.14	92.91	163.86
Switzerland	102.68	104.60	96.55	70.98	82.14	62.54
United Kingdom	126.22	128.57	111.97	81.95	100.97	159.36
United States	120.89	123.15	108.72	79.55	96.71	122.98

Note: MRR: Maximum Recovery Rate; TVR: Time-Varying Interest Rate; CATA: Model featuring catastrophes. In all three cases, α is set to 5%. In the case of a time-varying interest rate, maximum sustainable debt is computed assuming the interest rate prevailing in 2010. MPS: α is set to the historical maximum primary surplus, $\alpha = \max_t\{PS/Y\}$,

Table 4: It is not only Keynes's multiplier that counts (b_S vs. b_M); the 'sudden stop' in default can be quite costly (b_M vs. b_E).

Table 4: Maximum Static Borrowing, Maximum Sustainable Borrowing, and Equity-Like Measure with $\alpha = 5\%$ (%)

Country	b_S (1)	b_M (2)	b_E (3)
Australia	17.18	121.89	298.41
Austria	17.23	124.17	358.45
Belgium	17.00	113.52	292.79
Canada	16.75	103.09	266.57
Denmark	16.66	99.74	271.11
Finland	16.47	93.27	378.83
France	17.25	125.60	314.61
Germany	16.95	111.22	296.20
Greece	15.81	75.47	248.65
Hungary	16.58	96.80	317.09
Iceland	16.49	93.92	390.40
Ireland	17.33	129.77	2361.16
Italy	16.57	96.48	259.98
Korea	18.44	236.70	∞
Netherlands	17.09	117.65	286.73
New Zealand	16.21	85.62	198.99
Norway	17.68	152.75	419.81
Portugal	16.43	91.98	394.61
Spain	17.21	123.49	314.18
Sweden	16.64	99.02	266.04
Switzerland	16.28	87.67	188.78
United Kingdom	16.87	107.73	303.23
United States	16.75	103.19	272.26

Table 5: There is very little PD at MSD.

Table 5: Default Probability at Maximum Sustainable Debt $PD(d_M)$ (%)

Country	$\alpha = 5.00\%$ (1)	MRR (2)	TVR (3)	CATA (4)	$\alpha = 4.00\%$ (5)	MPS (6)
Australia	0.32	0.38	0.00	0.51	0.32	0.32
Austria	0.36	0.42	0.00	0.59	0.36	0.36
Belgium	0.36	0.43	0.00	0.61	0.36	0.36
Canada	0.41	0.50	0.00	0.71	0.41	0.41
Denmark	0.44	0.54	0.00	0.79	0.44	0.44
Finland	0.62	0.77	0.01	1.37	0.62	0.62
France	0.31	0.37	0.00	0.50	0.31	0.31
Germany	0.38	0.46	0.00	0.65	0.38	0.38
Greece	0.71	0.93	0.03	1.90	0.71	0.71
Hungary	0.53	0.65	0.00	1.03	0.53	0.53
Iceland	0.62	0.78	0.01	1.38	0.62	0.62
Ireland	0.57	0.67	0.00	1.19	0.57	0.57
Italy	0.46	0.57	0.00	0.83	0.46	0.46
Korea	0.81	0.89	0.01	2.77	0.81	0.81
Netherlands	0.33	0.39	0.00	0.54	0.33	0.33
New Zealand	0.46	0.58	0.00	0.83	0.46	0.46
Norway	0.27	0.31	0.00	0.41	0.27	0.27
Portugal	0.65	0.81	0.01	1.49	0.65	0.65
Spain	0.32	0.38	0.00	0.52	0.32	0.32
Sweden	0.44	0.54	0.00	0.79	0.44	0.44
Switzerland	0.41	0.51	0.00	0.71	0.41	0.41
United Kingdom	0.41	0.50	0.00	0.73	0.41	0.41
United States	0.41	0.51	0.00	0.72	0.41	0.41

Note: MRR: Maximum Recovery Rate; TVR: Time-Varying Interest Rate; CATA: Model featuring catastrophes. In all three cases, α is set to 5%. In the case of a time-varying interest rate, the default probability is computed assuming the interest rate prevailing in 2010. MPS: α is set to the historical maximum primary surplus, $\alpha = \max_t\{PS/Y\}$,

Table 6: There was even less PD in 2010

Table 6: Default Probability in 2010 $PD(d_{2010})$ (%)

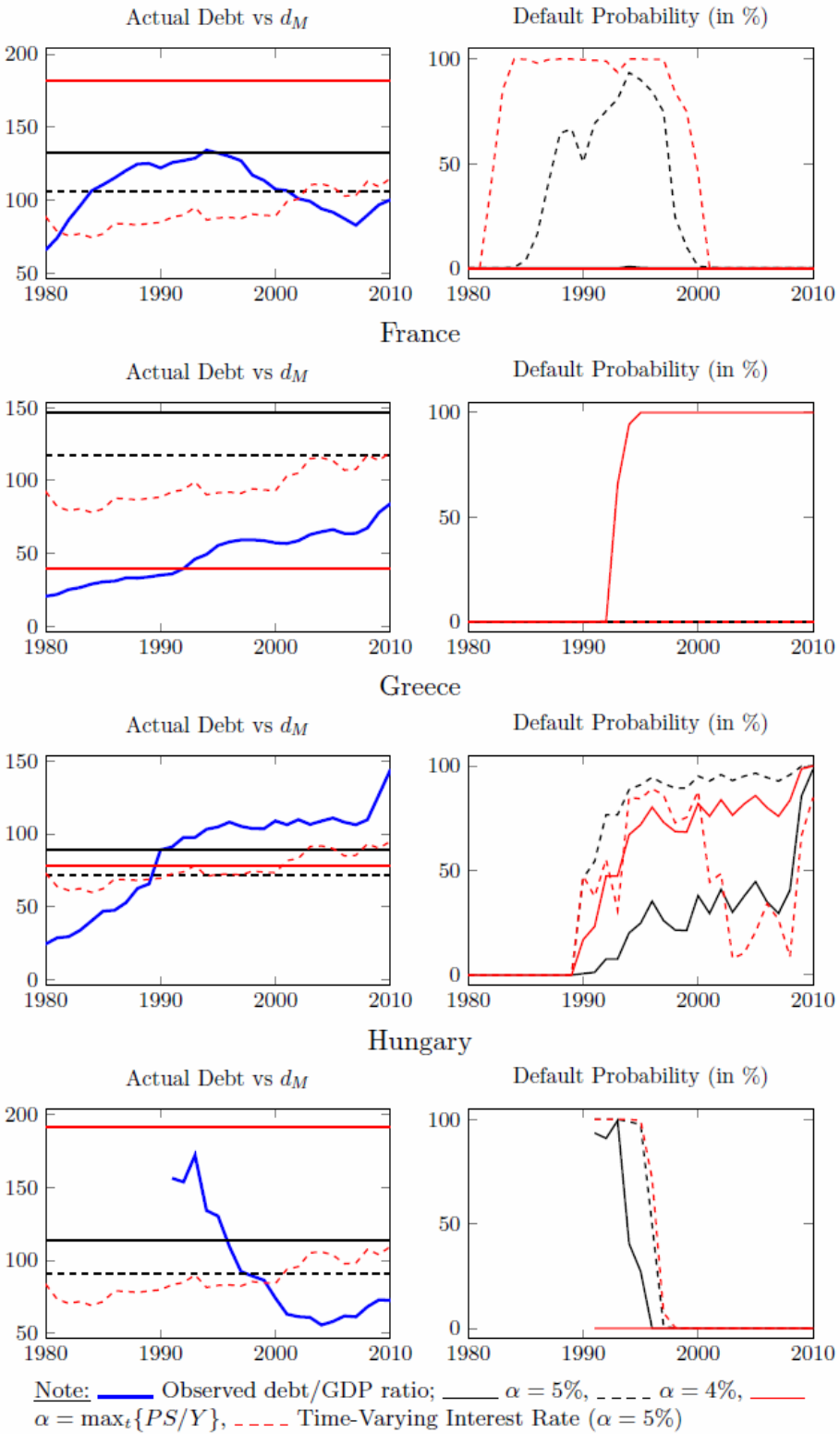
Country	$\alpha = 5.00\%$ (1)	MRR (2)	TVR (3)	CATA (4)	$\alpha = 4.00\%$ (5)	MPS (6)
Australia	0.00	0.00	0.00	0.00	0.00	0.00
Austria	0.00	0.00	0.00	0.00	0.00	0.00
Belgium	0.00	0.00	0.00	13.32	0.00	0.00
Canada	0.00	0.00	0.00	1.43	0.00	0.00
Denmark	0.00	0.00	0.00	0.00	0.00	0.00
Finland	0.00	0.00	0.00	0.00	0.00	0.00
France	0.00	0.00	0.00	0.00	0.00	100.00
Germany	0.00	0.00	0.00	0.00	0.00	0.00
Greece	98.33	97.25	85.50	100.00	100.00	99.94
Hungary	0.00	0.00	0.00	1.17	0.00	0.00
Iceland	2.62	1.24	0.15	98.84	71.76	0.00
Ireland	0.00	0.00	0.00	4.09	0.00	0.00
Italy	2.62	1.28	0.85	99.71	89.24	0.00
Korea	0.00	0.00	0.00	0.00	0.00	0.00
Netherlands	0.00	0.00	0.00	0.00	0.00	0.00
New Zealand	0.00	0.00	0.00	0.00	0.00	0.00
Norway	0.00	0.00	0.00	0.00	0.00	0.00
Portugal	0.00	0.00	0.00	22.40	0.09	100.00
Spain	0.00	0.00	0.00	0.00	0.00	0.00
Sweden	0.00	0.00	0.00	0.00	0.00	0.00
Switzerland	0.00	0.00	0.00	0.00	0.00	0.00
United Kingdom	0.00	0.00	0.00	0.03	0.00	0.00
United States	0.00	0.00	0.00	16.47	0.02	0.00

Note: MRR: Maximum Recovery Rate; TVR: Time-Varying Interest Rate; CATA: Model featuring catastrophes. In all three cases, α is set to 5%. In the case of a time-varying interest rate, the default probability is computed assuming the interest rate prevailing in 2010. MPS: α is set to the historical maximum primary surplus, $\alpha = \max_t\{PS/Y\}$,

Variation over Time

Figure 6: Fortunate France, profligate Greece, and virtuous Hungary

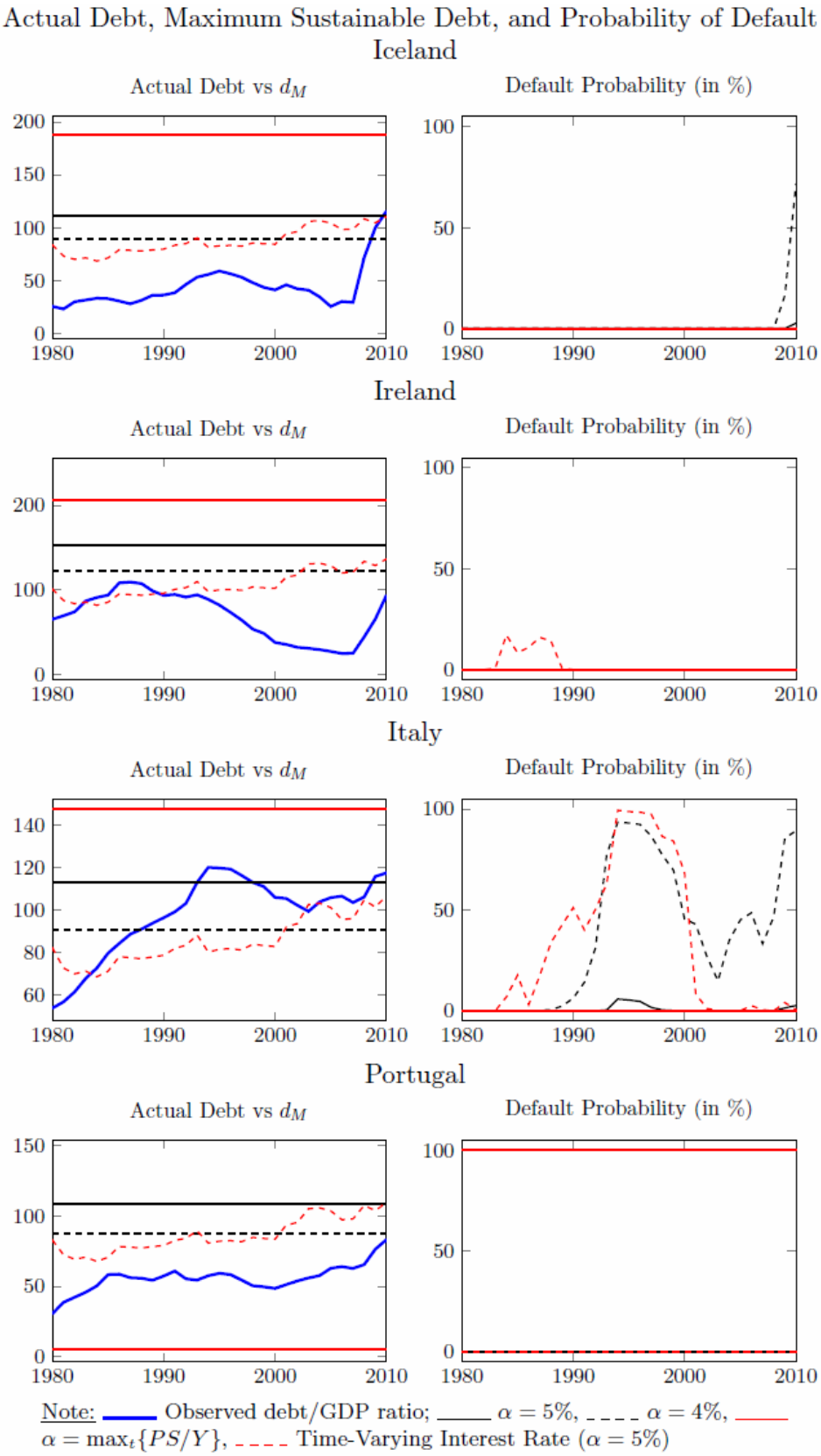
Figure 6: Actual Debt, Maximum Sustainable Debt, and Probability of Default over Time



Note: Observed debt/GDP ratio; $\alpha = 5\%$, $\alpha = 4\%$, $\alpha = \max_t\{PS/Y\}$, Time-Varying Interest Rate ($\alpha = 5\%$)

Figure 7: Problematic Iceland, Ireland, and (perhaps less so) Spain

Figure 7: Actual Debt, Maximum Sustainable Debt, and Probability of Default over Time



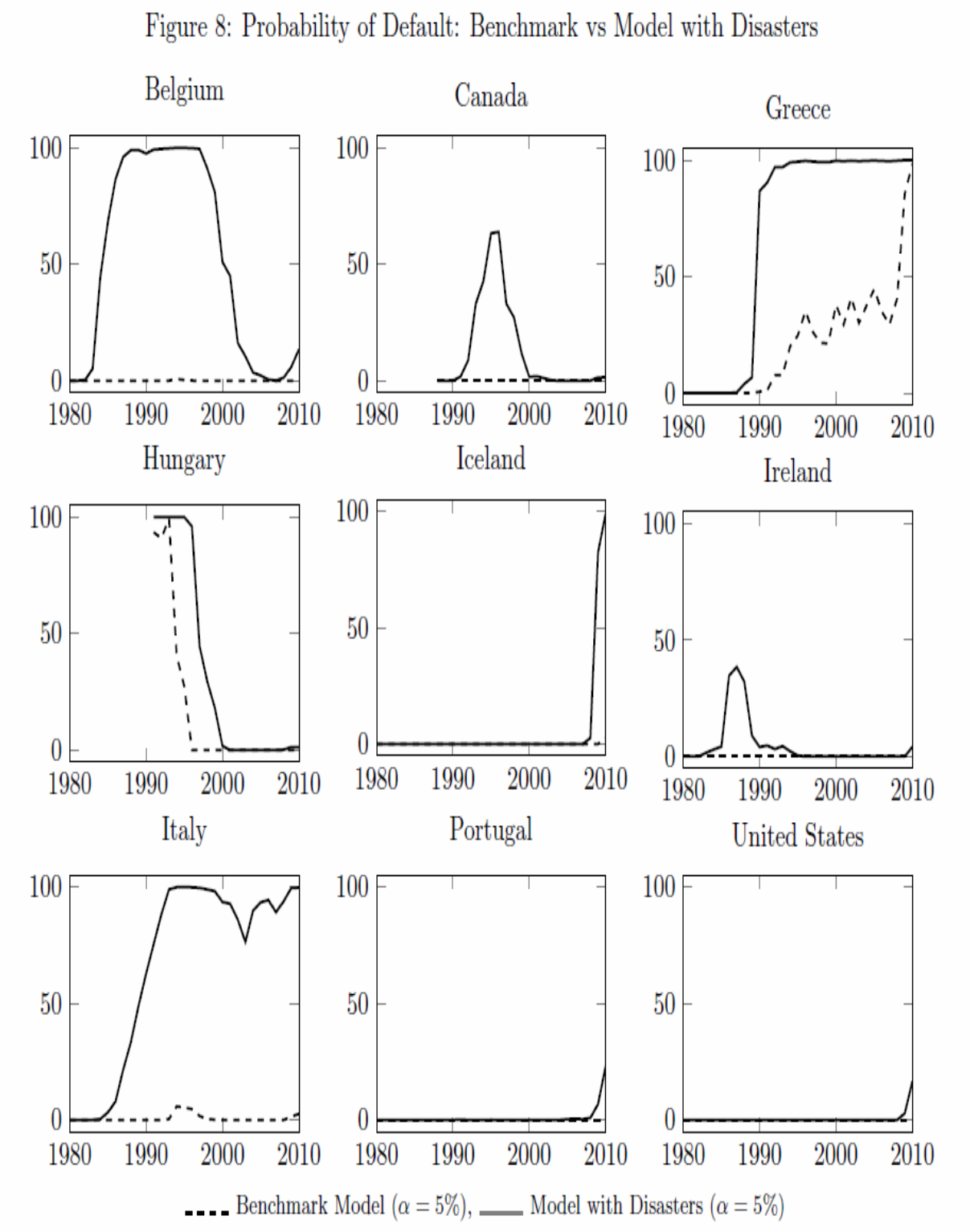
Extensions

Assuming 100% recovery in default makes very, very little difference to our results.

Assuming a time-varying risk-free rate decreases MSD, for governments recognize that high future interest rate realizations decrease future borrowing proceeds, thereby jeopardizing governments' ability to service debt.

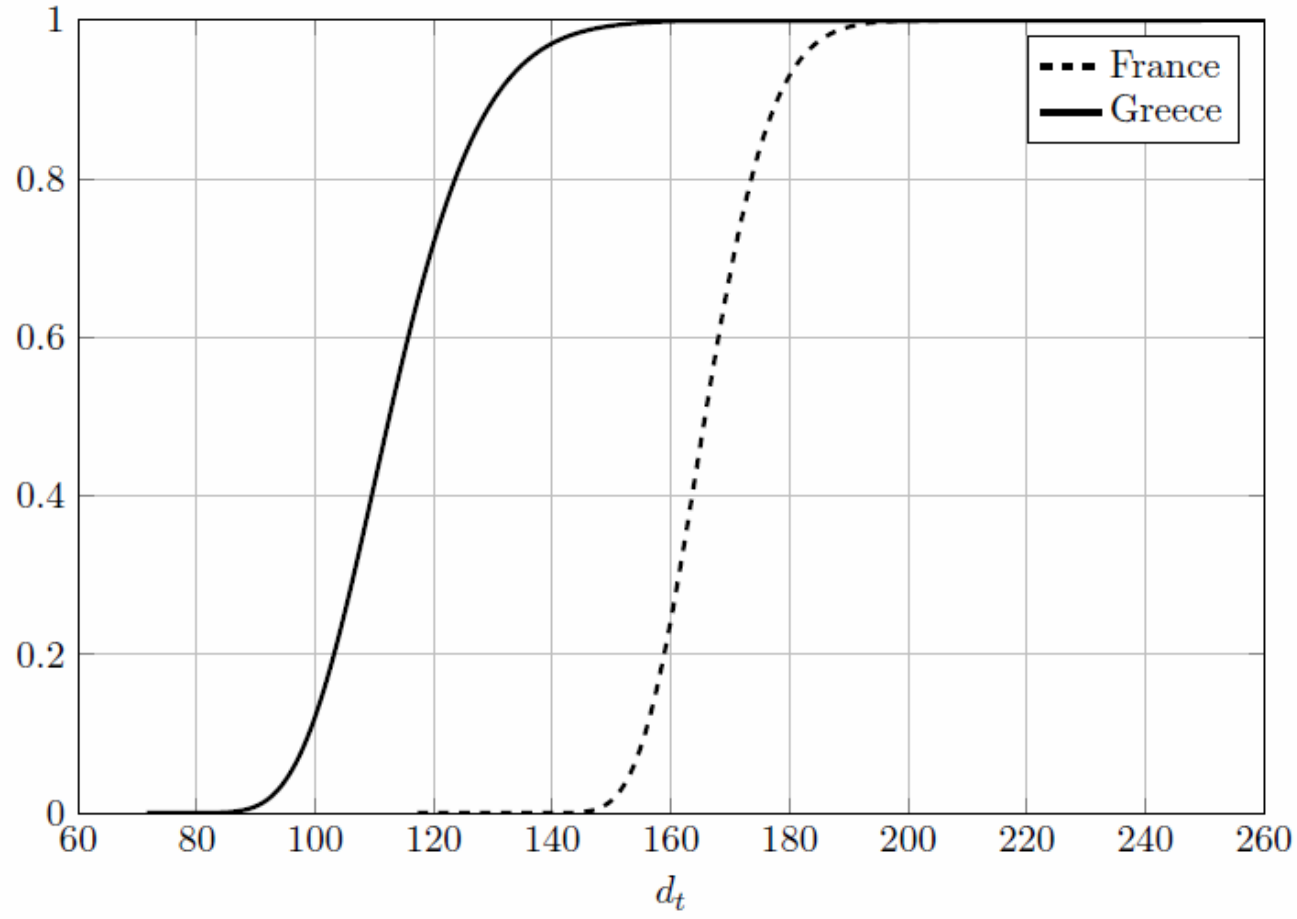
Allowing for the possibility of growth collapses results in dramatic declines in MSD.

Figure 8: Investors *may* have become more inclined to contemplate the possibility of collapses in growth.



Conclusion

Figure 3: Default Probability ($\alpha = 5\%$)



The road to default slopes gently before Maximum Sustainable Debt; it slopes (very) steeply after.

The gates of Hell are open night and day; Smooth the descent, and easy is the way; But, to return, and view the cheerful skies; In this, the task and mighty labor lies (Virgil, trans. J. Dryden).