The Role of Budget Constraints in the Theory of Rational Expectations, A Framework for Modelling and Discussing Financial Stability

The responsibility for opinions presented here rest solely with the authors and do not represent those of the Bank of Albania.

Outline

I. Objective & Motivation

II. Introduction

III. The models and assumptions

DSGE, VAR, Cointegration

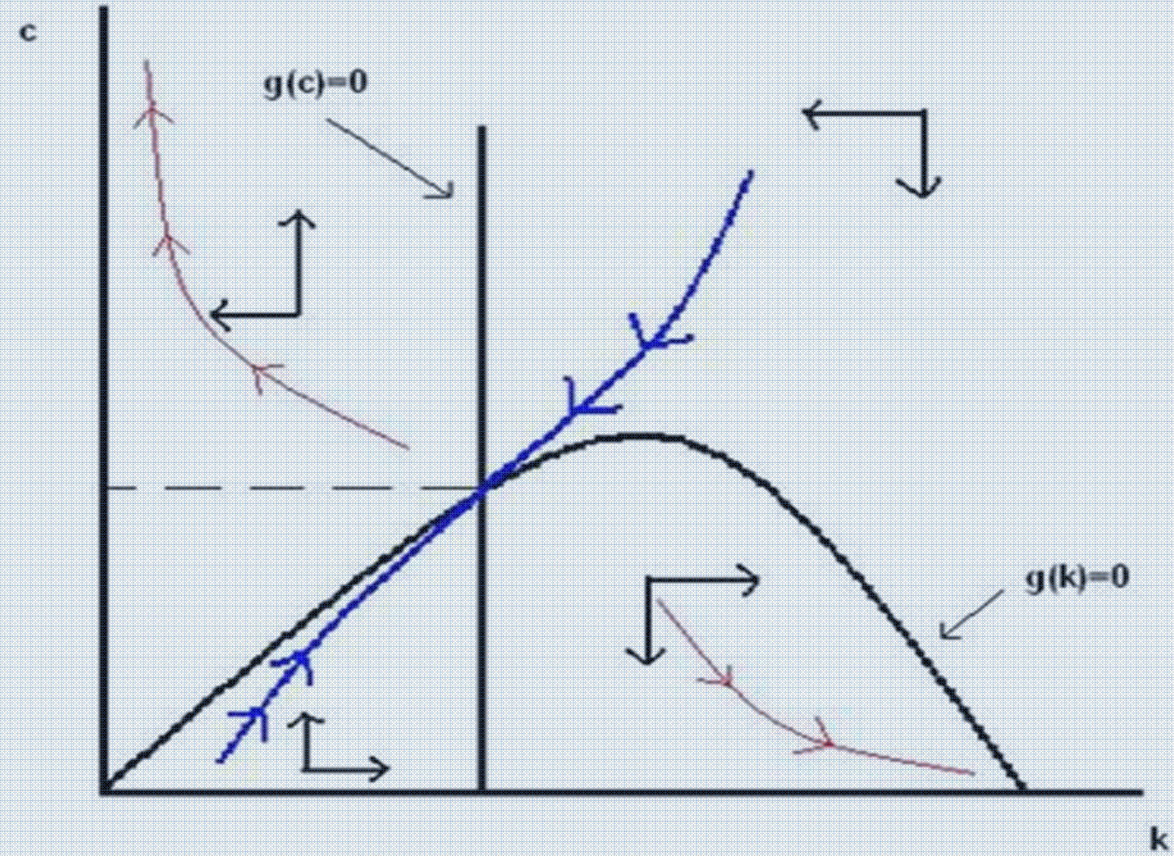
IV. Rational expectations and a new hybrid system

V. Conclusions

Objective & Motivation

- Rich vs. poor
  - *Why have reach economies converged ?*
  - *Why don't poor economies ?*

- Monetary policy Vs. Financial stability
  - *Which one comes first ?*
  - *Are they related ?*



The blue line represent the dynamic adjustment path of the economy. It is a stable path of the dynami system.
The red lines represent dynamic paths which are ruled out by the Transversality Condition

Introduction

- *The Real business cycle model*
  - *Hansen (1980)Models of economic behavior*
  - *Based on economic theory and microeconomic foundations*
- *Incorporate rational expectations*
  - *Blanchard and Kahn (1980)*
- *Bringing models to the data*
  - *Ireland (2004)*
- *The raise and fall of DSGE*
  - *Pre crisis state of the art models*
  - *Post crisis useless and unrealistic*
- *Alternative approach VAR*
  - *Explain the future from the past, Sims (1980)*

The Standard DSGE Model

- One representative agent that maximizes utility: $E\sum_{t=0}^{\infty}\beta^t\left[\ln(C_t)-\gamma H_t\right]$

$$C_t + I_t = Y_t = A_t K_{t-1}^{\alpha}\left(H_t\right)^{1-\alpha}$$

- Markets clear each period:
  - $K^S = K^D$
  - $H^S = H^D$
  - ***All markets are in equilibrium each period***

The Model In Per Capita Terms

- One representative agent that maximizes utility:

$$\max_{c_t, l_t} \quad E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_t^{1-\sigma}}{1-\sigma} - h_t^{\lambda} \right)$$

$$y_{t+1} = (1 + r_t) k_t + w_t h_t$$

$$k_{t+1} = i_t + (1 - \delta) k_t$$

$$y_t = c_t + i_t$$

$$y_t = a\, k_{t-1}^{\alpha} h_t^{1-\alpha}$$

$$a_t = \rho a_{t-1} + (1 - \rho) a^* + \varepsilon_t$$

Solving The Model

- It takes several steps to solve the model:
  - Find the steady state solution
    - Find first order conditions
  - The model is non linear
    - Log liberalize around the steady state
  - Solve the model recursively
  - Bring the model to data

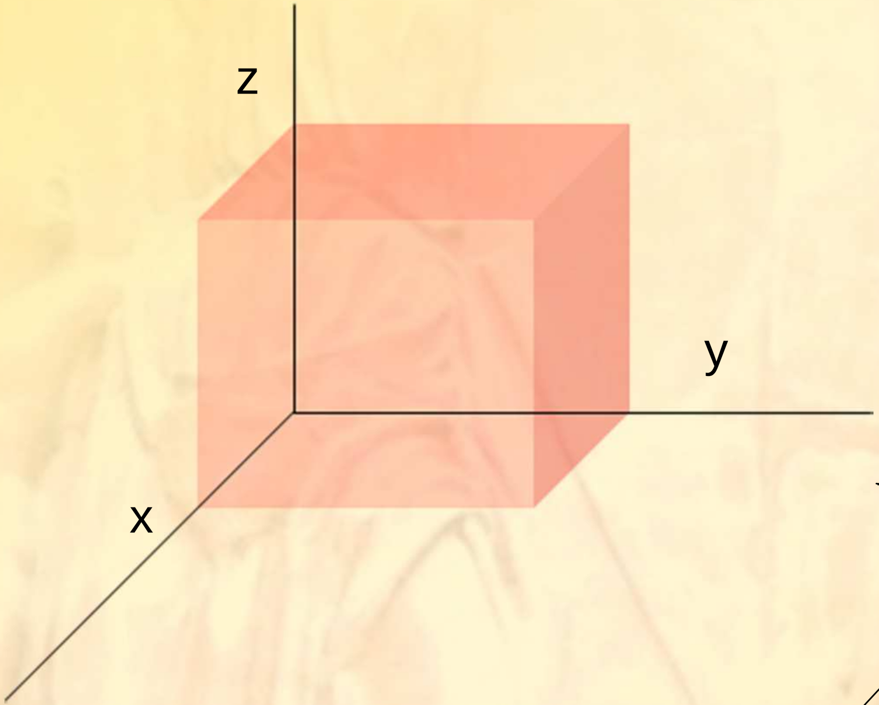
The complete log-linearized system

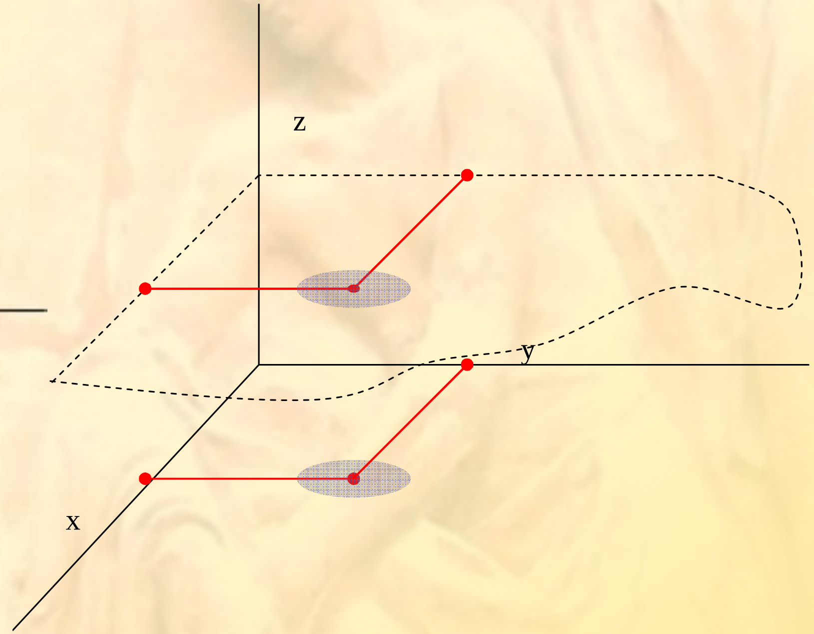
$$
\begin{aligned}
\hat{w}_t &= \hat{c}_t + \frac{N}{1-N}\hat{n}_t \\
\hat{c}_t &= \hat{c}_{t+1} - \frac{\rho}{1+\rho}\hat{r}_{t+1} \\
\hat{w}_t &= \hat{y}_t - \hat{n}_t \\
\hat{r}_t &= \frac{\rho+\delta}{\rho}\left(\hat{y}_t - \hat{k}_{t-1}\right) \\
\hat{y}_t &= \frac{C}{Y}\hat{c}_t + \frac{K}{Y}\left(\hat{k}_t + (1-\delta)\,\hat{k}_{t-1}\right) \\
\hat{y}_t &= \hat{a}_t + \alpha\hat{k}_{t-1} + (1-\alpha)\,\hat{n}_t
\end{aligned}
$$

plus the exogenous shock process

$$a_t = \rho_a a_{t-1} + \hat{a}_t$$

From R^3 to R^2





Difference Equations

Estimation

$$m_t = \omega_{11} m_{t-1} + \omega_{12} x_{t-1} + e_t^m$$
$$x_t = \omega_{21} m_{t-1} + \omega_{22} x_{t-1} + e_t^x$$

$$\mathbf{y}_t = \begin{pmatrix} m_t \\ x_t \end{pmatrix}$$

$$\mathbf{y}_{t-1} = \begin{pmatrix} m_{t-1} \\ x_{t-1} \end{pmatrix}$$

$$\mathbf{y}_t = \Omega \mathbf{y}_{t-1} + \mathbf{e}_t$$

$$\Omega = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{pmatrix}$$

Dynamics

$$\Delta e_t^m \rightarrow \Delta m_{t+l}$$
$$\Delta e_t^m \rightarrow \Delta x_{t+l}$$
$$\Delta e_t^x \rightarrow \Delta m_{t+l}$$
$$\Delta e_t^x \rightarrow \Delta x_{t+l}$$

$$l = 0, \ldots, \infty$$

$$\Delta \mathbf{e}_t \rightarrow \Delta \mathbf{y}_{t+l}$$

Matrix Form & R.E.

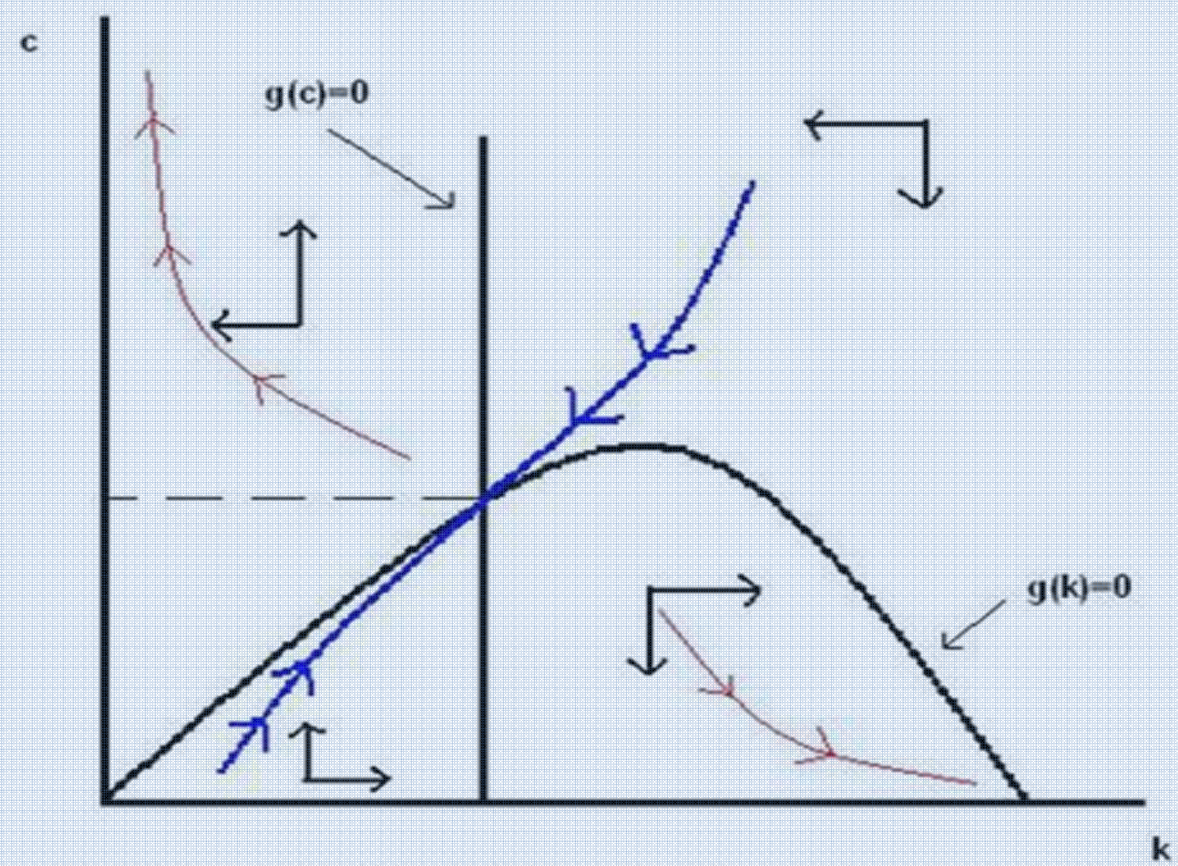
Blanchard and Kahn (1980)

$$y_t = \Omega y_{t-1}$$

$$\begin{bmatrix} c_t \\ l_t \\ k_t \\ a_t \end{bmatrix}_t = \begin{bmatrix} \omega_{11} & \dots & & \omega_{14} \\ \vdots & . & & \vdots \\ & & . & \\ \omega_{11} & \dots & & \omega_{44} \end{bmatrix} \begin{bmatrix} c_{t-1} \\ l_{t-1} \\ k_{t-1} \\ a_{t-1} \end{bmatrix}$$

$$\Omega = Q^{-1} V Q$$

$$\begin{bmatrix} Q_U \\ Q_S \end{bmatrix} y_t = \begin{bmatrix} \theta_1 & \cdots & & 0 \\ \vdots & \theta_2 & & \vdots \\ & & \theta_3 & \\ 0 & \cdots & & \theta_4 \end{bmatrix} \begin{bmatrix} Q_U \\ Q_S \end{bmatrix} y_{t+1}$$



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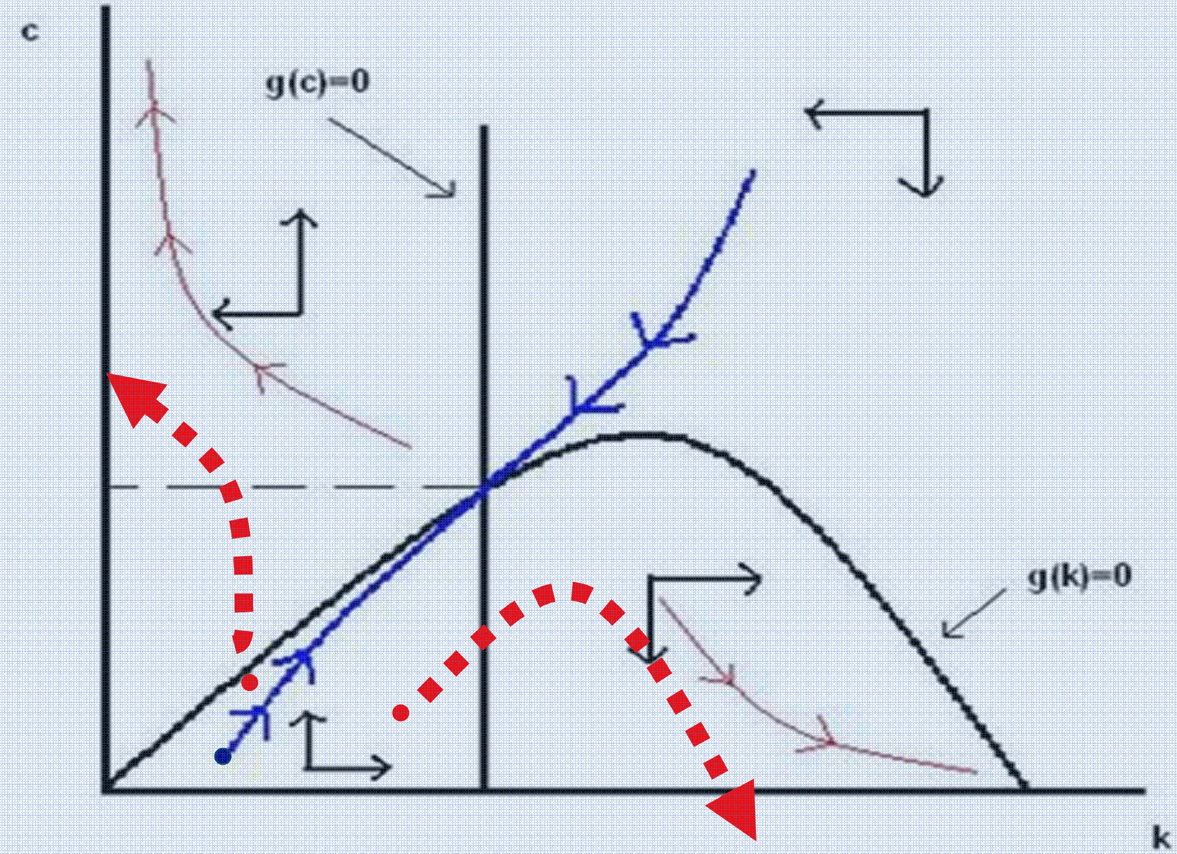

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Critique

- Unrealistic
  - Only one shock drives the economy
    - *Solved artificially by IRELAND (2004)*
  - Theoretic and sophisticated *SOLOW (2010)*
  - Failed to predict the crisis
    - Financial head winds (financial frictions) *Hall (2010)*
    - Capital requirement restrictions *Checcetti (2010)*
- Data does not support DSGE assumptions
  - Warlasian vs. Data – *Juselius, Francheti (2006)*

VAR

Sims 1980

Estimation

$$m_t = \omega_{11} m_{t-1} + \omega_{12} x_{t-1} + e_t^m$$
$$x_t = \omega_{21} m_{t-1} + \omega_{22} x_{t-1} + e_t^x$$

$$\mathbf{y}_t = \begin{pmatrix} m_t \\ x_t \end{pmatrix}$$

$$\mathbf{y}_{t-1} = \begin{pmatrix} m_{t-1} \\ x_{t-1} \end{pmatrix}$$

$$\mathbf{y}_t = \Omega \mathbf{y}_{t-1} + \mathbf{e}_t$$

$$\Omega = \begin{pmatrix} \omega_{11} & \omega_{12} \\ \omega_{21} & \omega_{22} \end{pmatrix}$$

Dynamics

$$\Delta e_t^m \rightarrow \Delta m_{t+l}$$
$$\Delta e_t^m \rightarrow \Delta x_{t+l}$$
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$$l = 0, \ldots, \infty$$

$$\Delta \mathbf{e}_t \rightarrow \Delta \mathbf{y}_{t+l}$$

Cointegration

Let $\mathbf{Y}_t = (y_{1t}, \ldots, y_{nt})'$ denote an $(n \times 1)$ vector of $I(1)$ time series. $\mathbf{Y}_t$ is *cointegrated* if there exists an $(n \times 1)$ vector $\boldsymbol{\beta} = (\beta_1, \ldots, \beta_n)'$ such that

$$\boldsymbol{\beta}'\mathbf{Y}_t = \beta_1 y_{1t} + \cdots + \beta_n y_{nt} \sim I(0)$$

In words, the nonstationary time series in $\mathbf{Y}_t$ are cointegrated if there is a linear combination of them that is stationary or $I(0)$.

- The linear combination $\boldsymbol{\beta}'\mathbf{Y}_t$ is often motivated by economic theory and referred to as a *long-run equilibrium* relationship.

- Intuition: $I(1)$ time series with a long-run equilibrium relationship cannot drift too far apart from the equilibrium because economic forces will act to restore the equilibrium relationship.

Cointegration and Error Correction Models

Consider a bivariate $I(1)$ vector $\mathbf{Y}_t = (y_{1t}, y_{2t})'$ and assume that $\mathbf{Y}_t$ is cointegrated with cointegrating vector $\boldsymbol{\beta} = (1, -\beta_2)'$ so that $\boldsymbol{\beta}'\mathbf{Y}_t = y_{1t} - \beta_2 y_{2t}$ is $I(0)$. Engle and Granger's famous (1987) *Econometrica* paper showed that cointegration implies the existence of an *error correction model* (ECM) of the form

$$\begin{aligned}
\Delta y_{1t} &= c_1 + \alpha_1(y_{1t-1} - \beta_2 y_{2t-1}) \\
&\quad + \sum_j \psi_{11}^j \Delta y_{1t-j} + \sum_j \psi_{12}^j \Delta y_{2t-j} + \varepsilon_{1t} \\
\Delta y_{2t} &= c_2 + \alpha_2(y_{1t-1} - \beta_2 y_{2t-1}) \\
&\quad + \sum_j \psi_{21}^j \Delta y_{1t-j} + \sum_j \psi_{22}^j \Delta y_{2t-j} + \varepsilon_{2t}
\end{aligned}$$

The ECM links the long-run equilibrium relationship implied by cointegration with the short-run dynamic adjustment mechanism that describes how the variables react when they move out of long-run equilibrium.

Assumptions by Juselius and Franchi

- Exogenity assumptions:
  - a_t & k_t drive the system
- Stationary assumptions
  - 1. y_t, c_t and k_t are trend stationary meaning that their combinations:
  - 2. y_t & k_t (1) and y_t & c_t (2) are stationary
  - 3. therefore h_t is stationary

Findings by Juselius and Franchi

- Exogenity assumptions
  - Capital is not weakly exogenous
- Stationary assumptions are not supported by data
  - Linear combination in1 and 2 are not stationary
  - h_t is not stationary
- Shocks to consumption are driving the system

All Markets Clear: SR vs.LR

All markets in clear in each period for all $t \in (0,+\infty)$

$$C_t + I_t = Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

All markets clear in the long run, but for some $t \in (0,+\infty)$

$$C_t + I_t \neq Y_t = A_t K_t^{\alpha} H_t^{1-\alpha}$$

$$(C_t + I_t) - Y_t = B_t$$

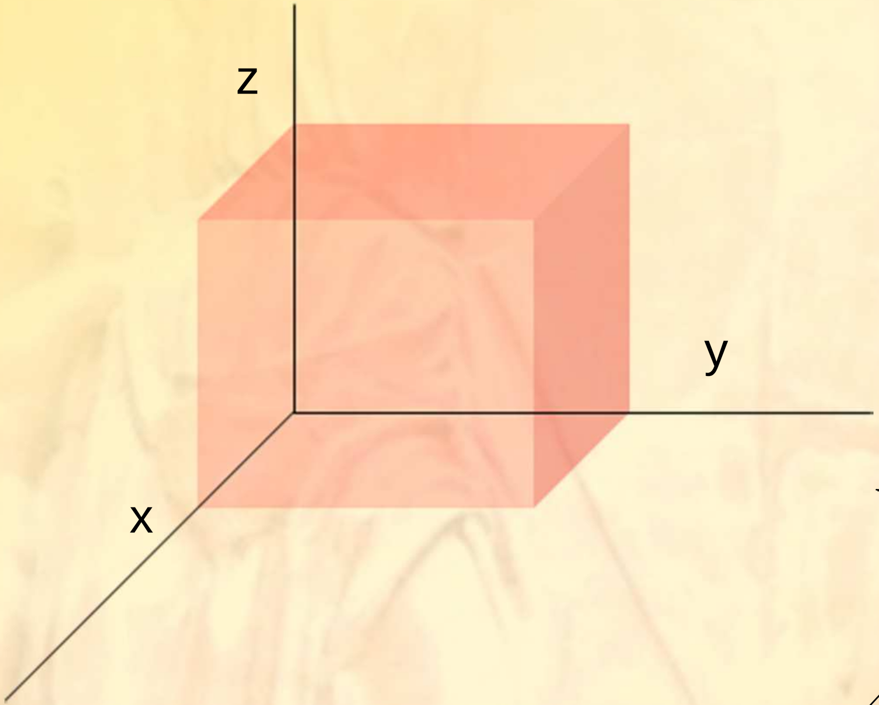
$$\lim_{t \to \infty} B_t = 0$$

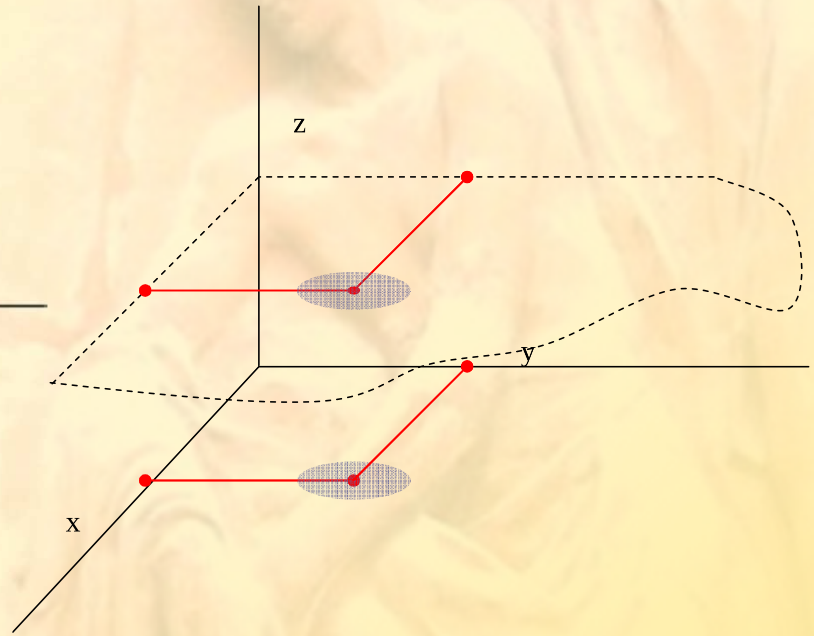
c, y & k are Cointegrated in the long run

$$c^* + \delta k^* = y^*$$

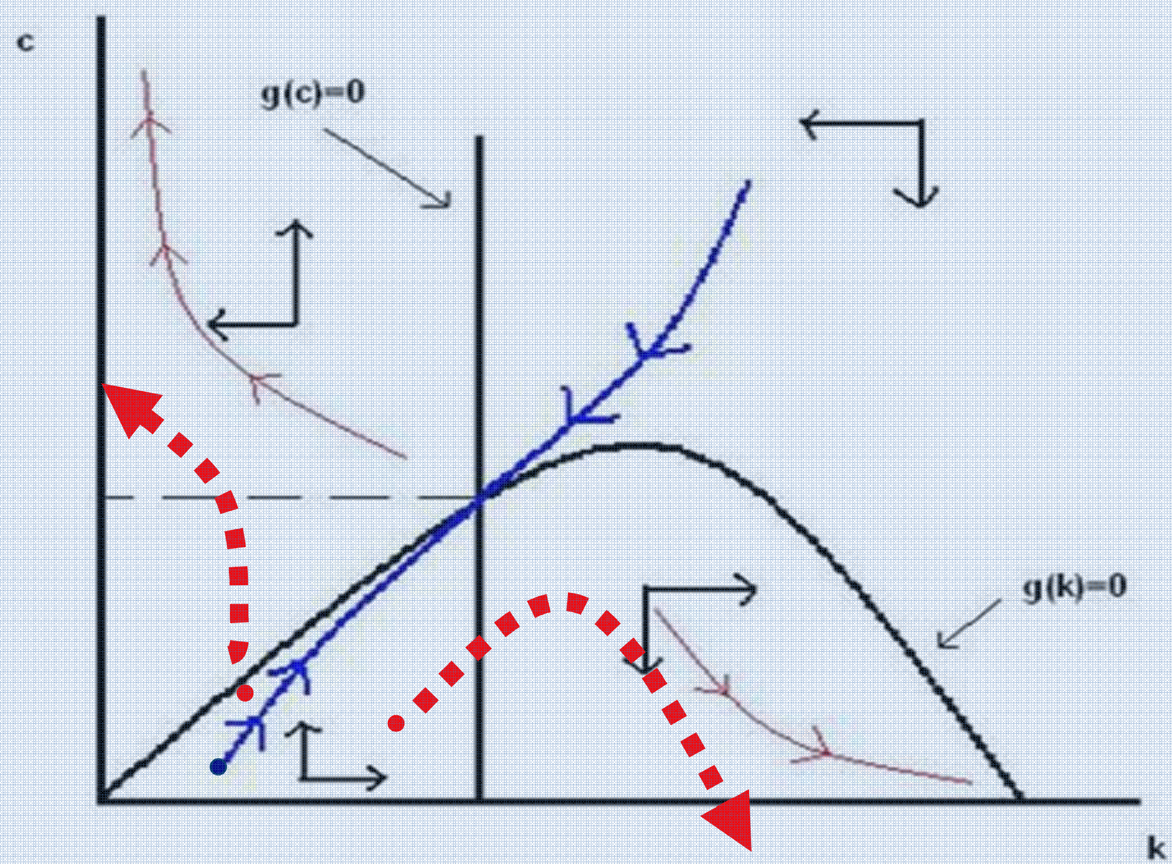

Financial gap
time t→ ∞

From R^n to R^m and vice versa





for n = 3 and m = n-1 = 2



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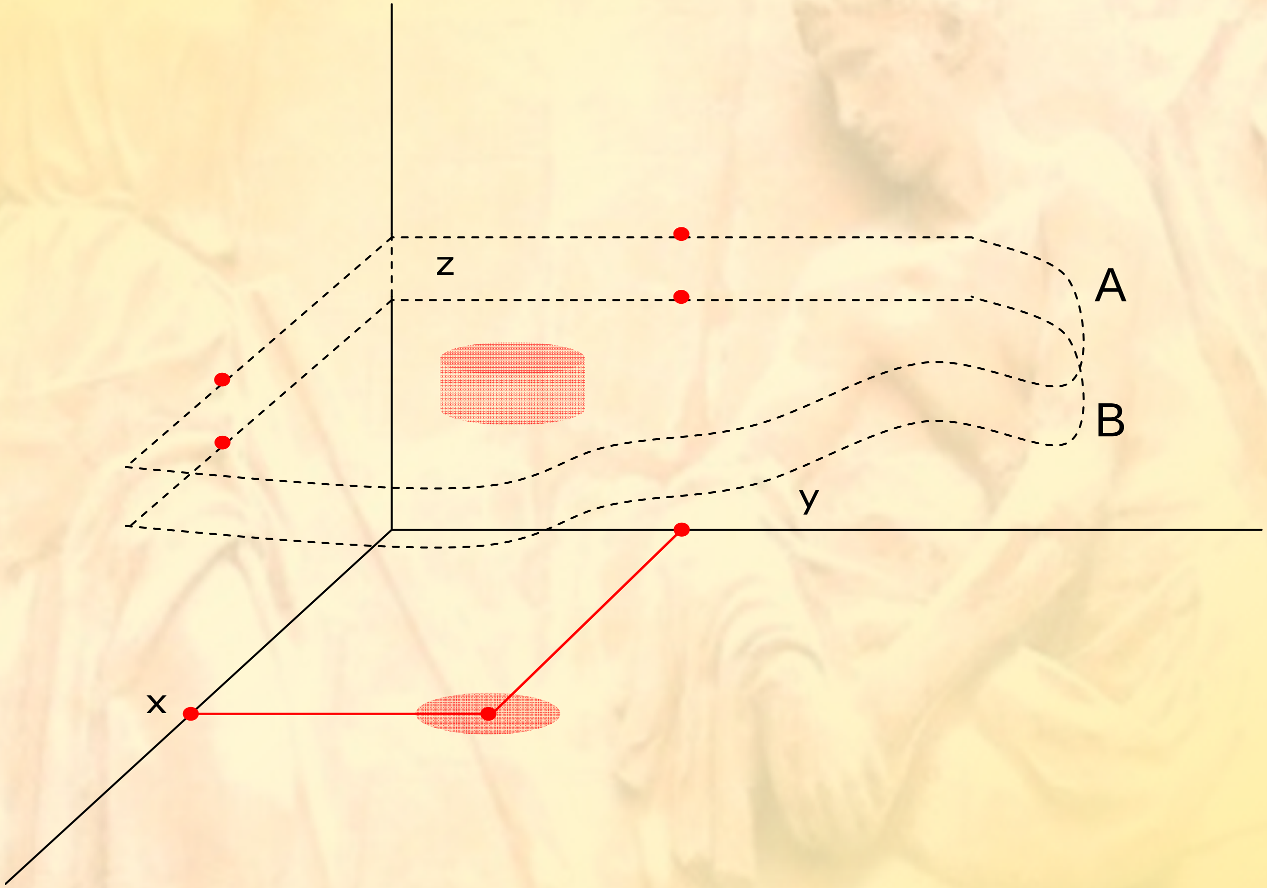
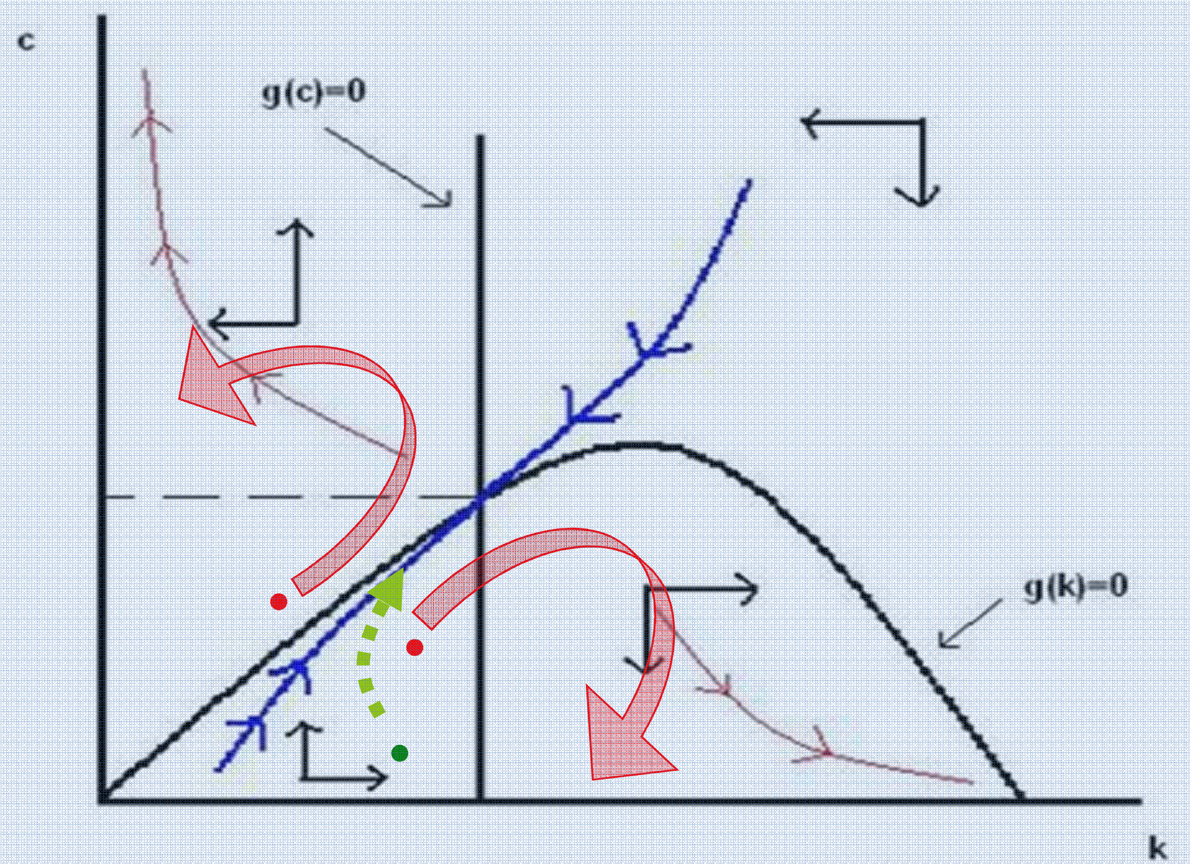


New Matrix Form & R.E.

$$
y_t \quad \begin{bmatrix} c_t \\ l_t \\ \hline k_t \\ a_t \end{bmatrix}_t = \left[\begin{array}{cc|cc} \omega_{11} & \cdots & & \omega_{14} \\ \vdots & . & & \vdots \\ \hline & & . & \\ \omega_{11} & \cdots & & \omega_{44} \end{array}\right] \begin{bmatrix} c_{t+1} \\ l_{t+1} \\ \hline k_{t+1} \\ a_{t+1} \end{bmatrix} \quad y_{t+1}
$$



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Expected Benefits

- Allows for Financial Stability Analysis
  - *The presence of ECM brings stability in the model*
- More flexibility of the model
  - *It allows for irrational choices in the short run*
  - *Long run rationality vs. rationality at each period*
- Increase the possible number of shocks in the model
  - *Introduce as many shocks in the model as there are variables in the cointegration space.*

Thank you