The Real Exchange Rate, Real Interest Rates, and the Risk Premium

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How does exchange rate respond to interest rate changes?

- In standard open-economy New Keynesian model, increase in interest rate leads to real appreciation
- Under uncovered interest parity (UIP) , currency then depreciates
- But famous UIP puzzle says when interest rate goes up, currency expected to appreciate (but says nothing about the level)
- What is actual behavior of level and change? Can models explain it?
- Important for monetary policy:
  - First, just to understand impact of policy
  - Second, to understand economic forces at work

Definition of Terms

--(Log of) nominal exchange rate: s

(Price of Foreign currency – increase means Home depreciation)

--(Log) of Home and Foreign prices: p, p^*

-- Home and Foreign nominal interest rates: i, i^*

-- Home and Foreign inflation: $\pi_{t+1} = p_{t+1} - p_t$, $\pi^*_{t+1} = p^*_{t+1} - p^*_t$

--(Log of) real exchange rate: $q = s + p^* - p$

(Increase means Home real depreciation)

-- Home and Foreign real interest rates: $r_t = i_t - E_t\pi_{t+1}$, $r^*_t = i^*_t - E_t\pi^*_{t+1}$

Uncovered Interest Parity: $i^*_t + E_t s_{t+1} - s_t = i_t$

Equivalently (adding $E_t\pi^*_{t+1} - E_t\pi_{t+1}$ to each side): $r^*_t + E_t q_{t+1} - q_t = r_t$

"Risk premium": $\lambda_t \equiv i^*_t + E_t s_{t+1} - s_t - i_t = r^*_t + E_t q_{t+1} - q_t - r_t$

Standard models assume UIP: $r_t^* + E_t q_{t+1} - q_t = r_t$

Rearrange and subtract off unconditional means:

$$q_t = -(r_t - r_t^* - \overline{r}) + E_t q_{t+1}$$

Iterate forward:
$$\begin{aligned} q_t &= -(r_t - r_t^* - \overline{r}) - E_t(r_{t+1} - r_{t+1}^* - \overline{r}) + E_t q_{t+2} \\ &= \ldots = -R_t + \lim_{j\to\infty} E_t q_{t+j} \end{aligned}$$

where $\quad R_t \equiv \sum_{i=0}^{\infty} E_t(r_{t+i}^* - r_{t+i} - \overline{r})$

Now assume also long run purchasing power parity: $\lim_{j\to\infty} E_t q_{t+j} = \overline{q}$

Then $q_t - \overline{q} = -R_t$

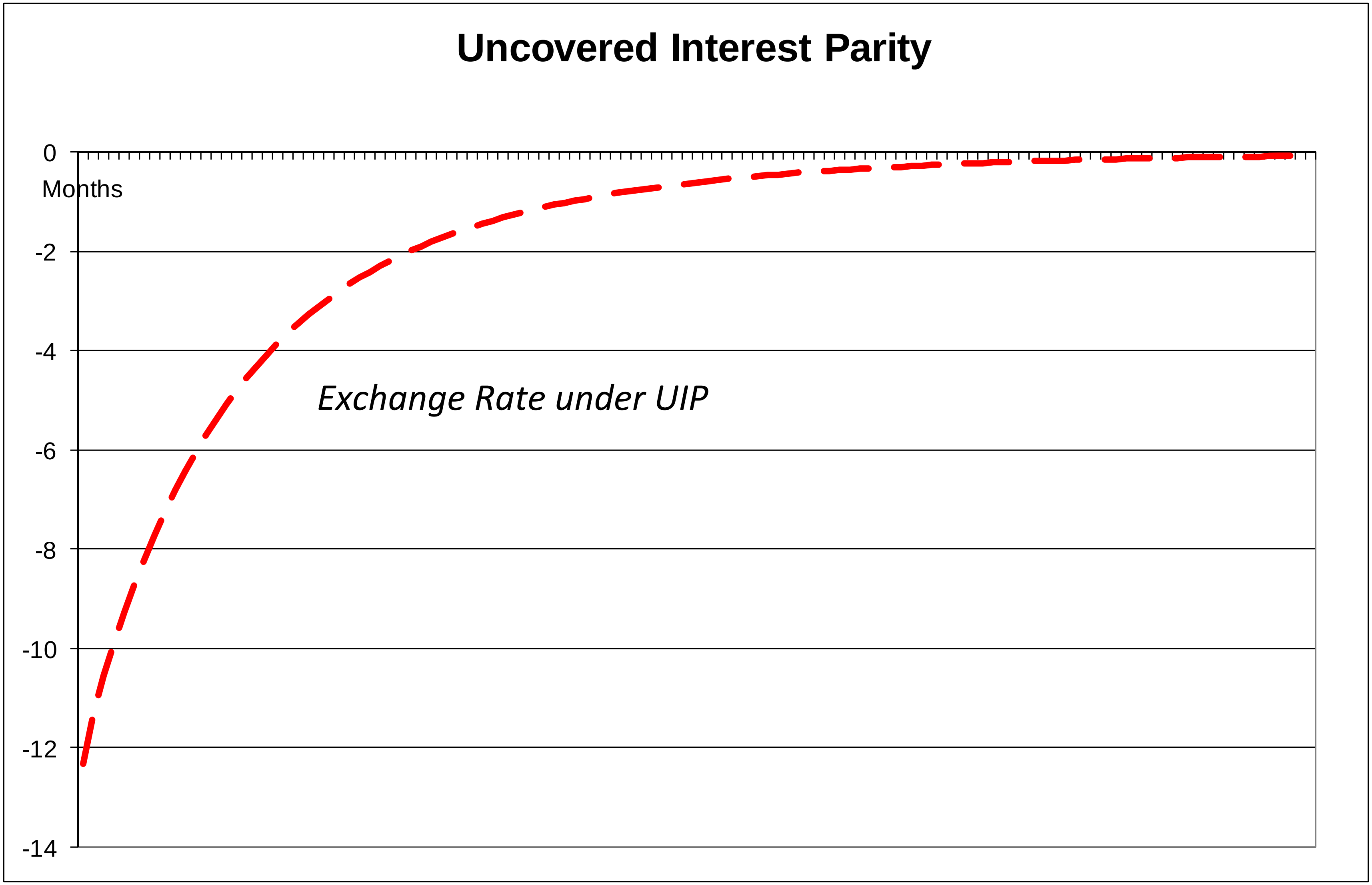
Interest rate differentials are positively serially correlated (AR1 for example).

When $r_t \uparrow$, we find $R_t \uparrow$

Under UIP, $q_t - \bar{q} = -R_t$, so when $r_t \uparrow$, we find $q_t \downarrow$

The UIP model is consistent with one well-known fact:

When a country's real interest rate moves up, its currency tends to strengthen in real terms.


Uncovered Interest Parity
0
Months
-2
-4
-6
-8
-10
-12
-14
Exchange Rate under UIP

A second well-known "facts" in international finance:

The forward-premium puzzle: Country with high interest rate has high excess return.

Regression: $s_{t+1} - s_t = \beta_0 + \beta_1(i_t - i_t^*) + u_{t+1}$.

UIP says $\beta_0 = 0$ and $\beta_1 = 1$.

Typical finding: $\beta_1 < 1$ (often $\beta_1 < 0$), $\beta_0 = 0$.

This means $\lambda_t \equiv i_t^* + E_t s_{t+1} - s_t - i_t = r_t^* + E_t q_{t+1} - q_t - r_t$ negatively correlated with $i_t - i_t^*$.

Paper shows that λ_t is negatively correlated with $r_t - r_t^*$ also.

(!!!!) Risk premium models have: $\lambda_t = -\gamma(r_t - r_t^*)$, $\gamma > 1$

To restate things:

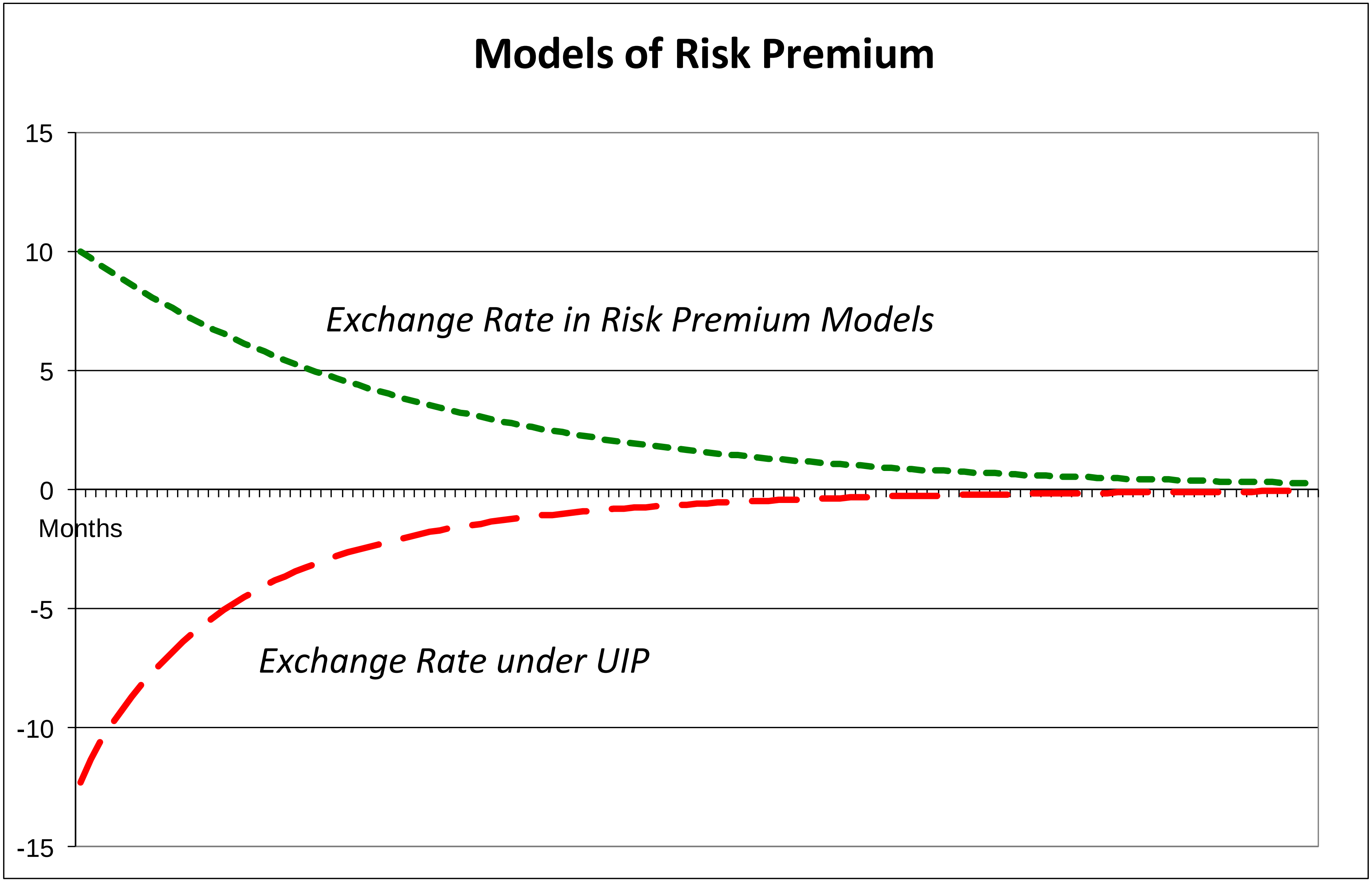
$E_t q_{t+1} - q_t = r_t - r_t^* + \lambda_t$.

How do you explain $E_t q_{t+1} - q_t \downarrow$ when $r_t - r_t^* \uparrow$?

In the models in the literature, $\lambda_t = -\gamma(r_t - r_t^*)$, $\gamma > 1$.

But what does this imply about how q_t changes when $r_t - r_t^* \uparrow$?

It implies $q_t - \bar{q} = (\gamma - 1)R_t$ so $q_t \uparrow$ when $r_t - r_t^* \uparrow$


Models of Risk Premium
15
10
5
0
-5
-10
-15
Months
Exchange Rate in Risk Premium Models
Exchange Rate under UIP

Here is the problem:

Most macro models central banks use assume UIP:

When $r_t - r_t^* \uparrow \Rightarrow q_t \downarrow$ (True)

When $r_t - r_t^* \uparrow \Rightarrow E_t q_{t+1} - q_t \uparrow$ (False)

Models that are used to explain UIP puzzle:

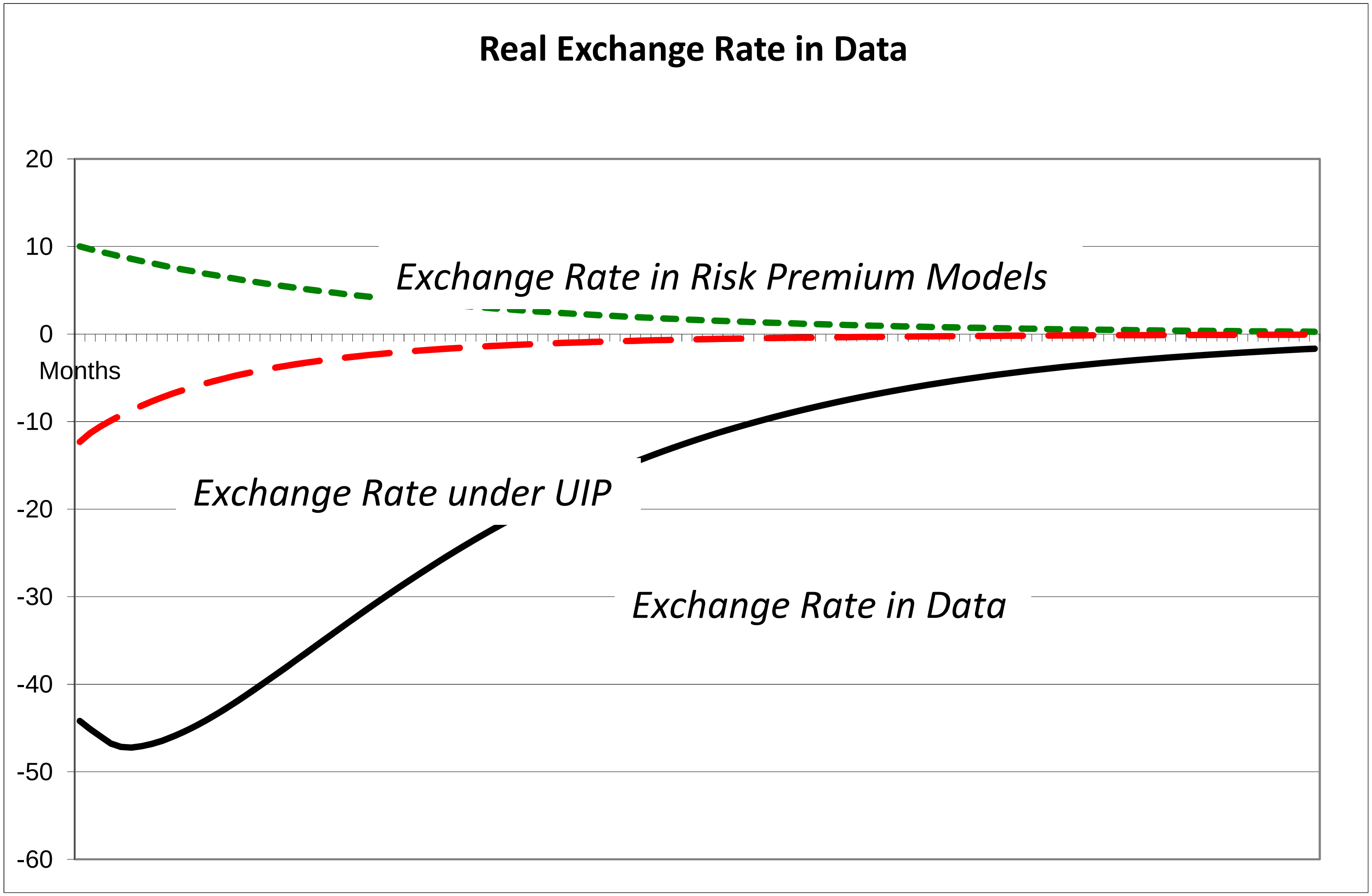
When $r_t - r_t^* \uparrow \Rightarrow q_t \uparrow$ (False)

When $r_t - r_t^* \uparrow \Rightarrow E_t q_{t+1} - q_t \downarrow$ (True)

What do the data say?

When $r_t - r_t^* \uparrow \Rightarrow q_t \downarrow$

When $r_t - r_t^* \uparrow \Rightarrow E_t q_{t+1} - q_t \downarrow$


Real Exchange Rate in Data
20
10
0
-10
-20
-30
-40
-50
-60
Months
Exchange Rate in Risk Premium Models
Exchange Rate under UIP
Exchange Rate in Data

Real interest rates and real exchange rates.

Rewrite: $q_t - E_t q_{t+1} = -(r_t - r_t^* - \bar{r}) - (\lambda_t - \bar{\lambda})$

Iterate forward to get:

$$q_t - \lim_{j\to\infty}\left(E_t q_{t+j}\right) = -R_t - \Lambda_t$$

where

$$R_t \equiv \sum_{j=0}^{\infty} E_t(r_{t+j} - r_{t+j}^* - \bar{r}) \qquad \Lambda_t \equiv \sum_{j=0}^{\infty} E_t(\lambda_{t+j} - \bar{\lambda})$$

R_t - "prospective real return differential"
Λ_t - "level risk premium"

We find when $r_t - r_t^* \uparrow$ that $E_t q_{t+1} - q_t \downarrow$ (implies $\lambda_t \downarrow$) but $q_t \downarrow$ and $\Lambda_t \uparrow$

What model accounts for this behavior? I don't know…

When $r_t - r_t^* \uparrow$ that $E_t q_{t+1} - q_t \downarrow$ (implies $\lambda_t \downarrow$) but $q_t \downarrow$ and $\Lambda_t \uparrow$

<u>Risk Premium?</u>

Overall Home short-term deposits expected to be less risky ($\Lambda_t \uparrow$)

But in the short run they are more risky ($\lambda_t \downarrow$)

None of our models can account for this.

<u>Liquidity Premium?</u>

Same problem – short-term and long-term premiums move in opposite directions.

<u>Market "Dynamics"</u>

Overreaction plus herding.

Understanding this is a challenge – but it matters!

Figure 1 plots slope coefficients from the following regressions (Data are monthly, interest rates are 1-month, end-of-month. For this slide, U.S. relative to weighted average of rest of G7):

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*)$$

$$-R_{t+k} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*)$$

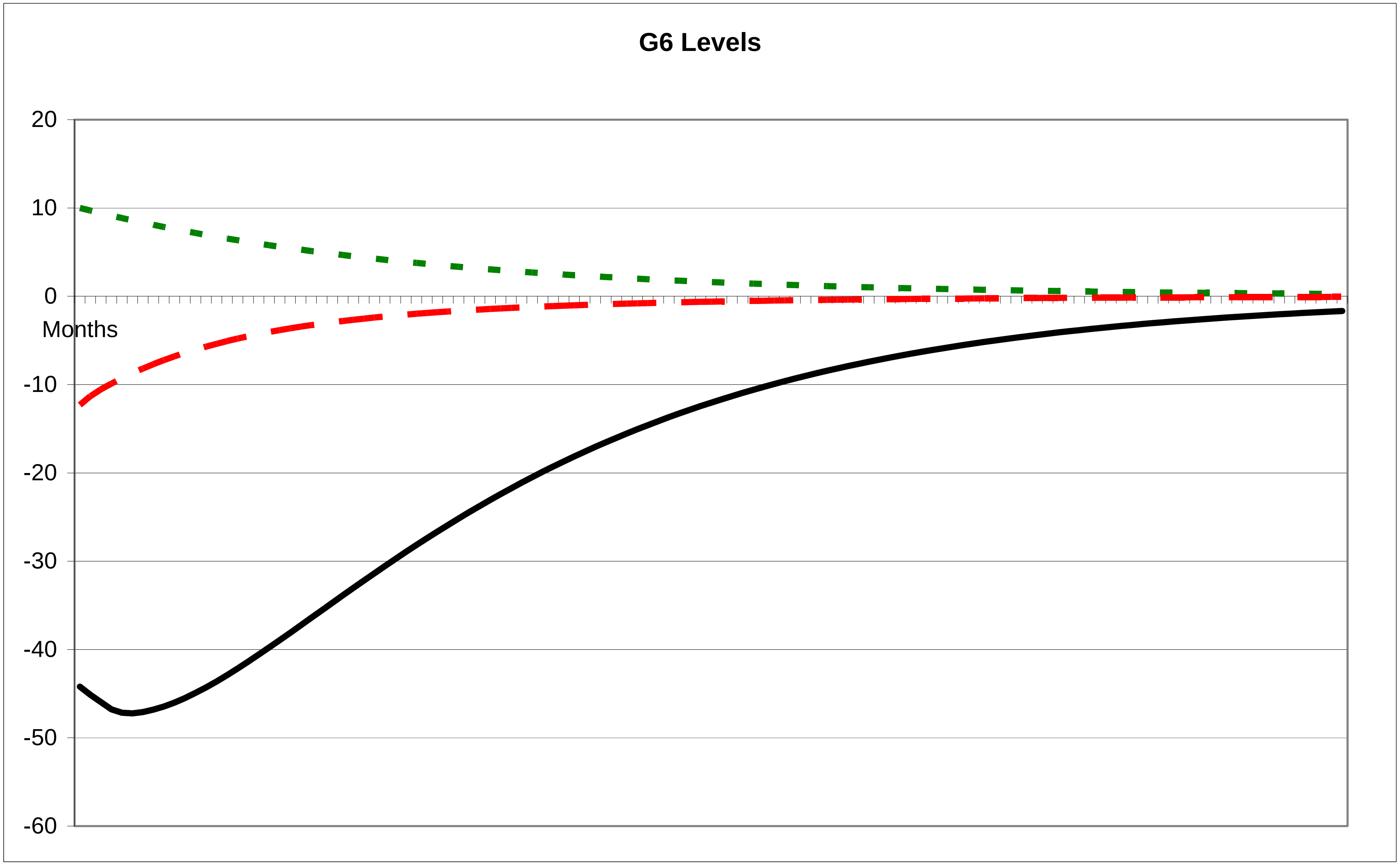
(If UIP held, we'd have $q_{t+k} = -R_{t,k}$.)

(Real interest rates themselves are estimates)

Difference between $-R_{t+k}$ and q_{t+k} is Λ_{t+k}:

$$\Lambda_{t+k} = -R_{t+k} - q_{t+k}.$$

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*),\ -R_{t+k} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*),\text{ and Model } q_{t+k} - \lim_{j\to\infty} E_t q_{t+j}$$



Data

U.S., Canada, France, Germany, Italy, Japan, U.K., and "G6"
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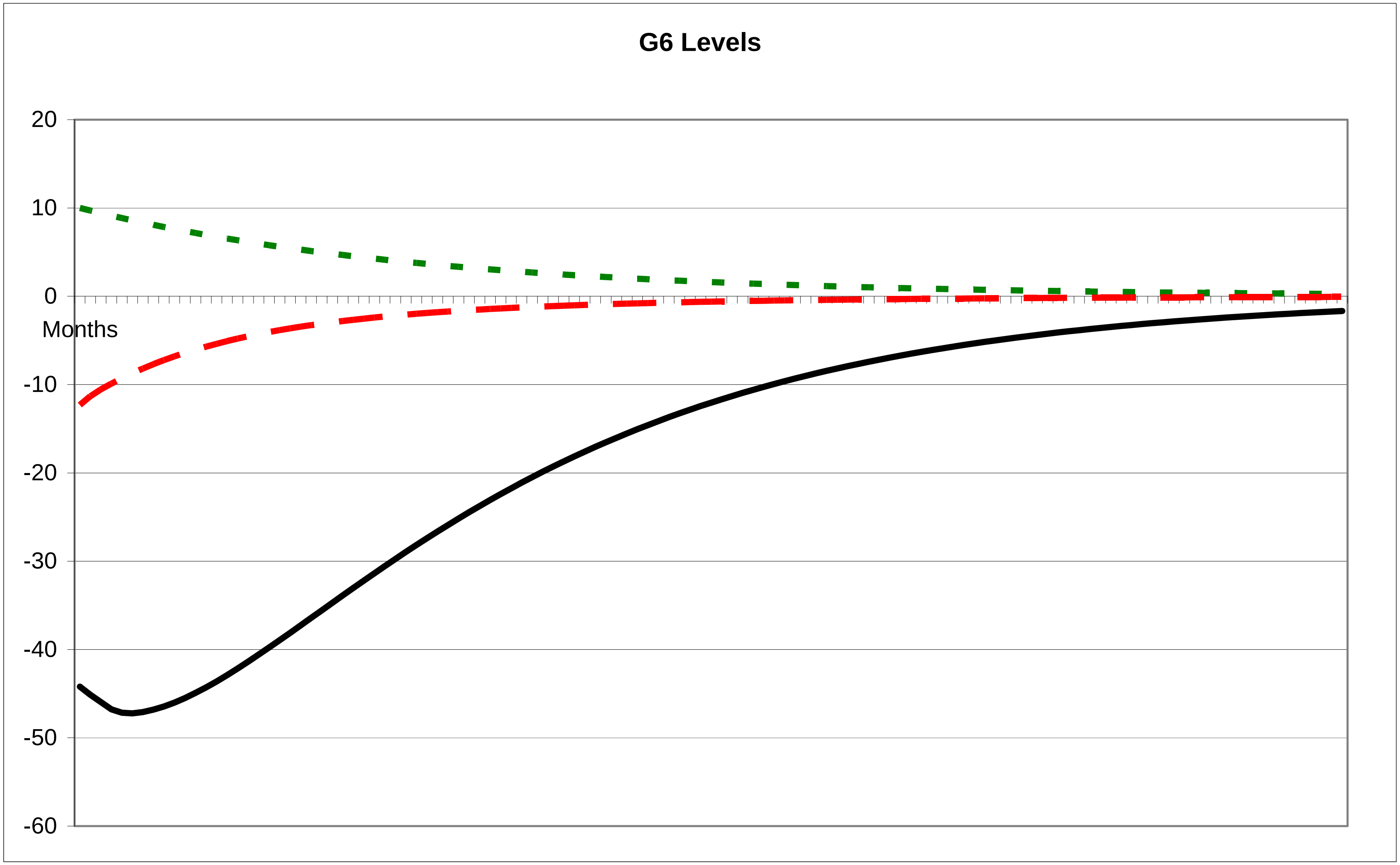

Exchange rates – last day of month (noon buy rates, NY)
Prices – consumer price indexes
Interest rates – 30-day Eurodeposit rates (last day of month)

Monthly, June 1979 – October 2009

(Unit root test for real exchange rates, uses data back to June, 1973)

Preliminary – unit root tests

Country	*ADF*	*DF-GLS*
Canada	-1.771	-1.077
France	-2.033	-2.036*
Germany	-2.038	-2.049*
Italy	-1.888	-1.914†
Japan	-2.071	-0.710
United Kingdom	-2.765†	-2.076*
G6	-2.052	-1.846†

Panel Unit Root Test, 1973:3-2009:10

Model	*Estimated Coefficient*	*1%*	*5%*	*10%*
No Covariates	-0.01705*	-0.02199	-0.01697	-0.01485
With Covariates	-0.01688*	-0.02126	-0.01638	-0.01411

Fama Regressions: $s_{t+1} - s_t = \beta_0 + \beta_1(i_t - i_t^*) + u_{t+1}$

1979:6-2009:10

Country	$\hat{\beta}_1$	*90% c.i.*($\hat{\beta}_1$)
Canada	-1.271	(-2.564,0.220)
France	-0.216	(-1.298,0.866)
Germany	-1.091	(-2.335,0.153)
Italy	0.661	(-0.206,1.528)
Japan	-2.713	(-4.141,-1.285)
U.K.	-2.198	(-3.646,-0.750)
G6	-1.467	(-2.783,-0.151)

VAR methodology

Two different VAR models:

Model 1: $\left[q_t, i_t - i_t^*, i_{t-1} - \pi_t - (i_{t-1}^* - \pi_t^*) \right]$

Model 2: $\left[q_t, i_t - i_t^*, \pi_t - \pi_t^* \right]$

Estimate VAR with 3 lags.

Use standard projection measures to estimate $r_t = i_t - E_t \pi_{t+1}$, $r_t^* = i_t^* - E_t \pi_{t+1}^*$, and $R_t \equiv \sum_{j=0}^{\infty} E_t (r_{t+j} - r_{t+j}^* - \overline{r})$

Then λ_t is constructed as $\lambda_t \equiv r_t^* + E_t q_{t+1} - q_t - r_t$

Λ_t estimate is constructed from $\Lambda_t = -R_t - q_t$

Fama Regression in Real Terms: $q_{t+1} - q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

Country	$\hat{\beta}_1$	*90% c.i.($\hat{\beta}_1$)*
Canada	0.138	(-1.222,1.498)
		(-1.908,1.632)
		(-1.800,1.676)
France	-0.576	(-2.269,1.117)
		(-2.240,0.719)
		(-2.602,1.125)
Germany	-0.837	(-2.689,1.015)
		(-3.458,0.313)
		(-3.419,0.411)
Italy	0.640	(-1.056,2.336)
		(-1.136,2.087)
		(-1.328,2.358)
Japan	-1.314	(-2.860,0.232)
		(-3.300,0.254)
		(-3.441,0.379)
United Kingdom	-1.448	(-3.042,0.146)
		(-3.614,-0.127)
		(-3.846,-0.039)
G6	-0.933	(-2.548,0.682)
		(-2.932,0.409)
		(-3.005,0.527)

Regression of q_t on $\hat{r}_t - \hat{r}_t^*$: $q_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

Country	$\hat{\beta}_1$	*90% c.i.($\hat{\beta}_1$)*
Canada	-48.517	(-62.15,-34.88)
		(-94.06,-31.41)
		(-140.54,-27.34)
France	-20.632	(-32.65,-8.62)
		(-44.34,-1.27)
		(-54.26,1.75)
Germany	-52.600	(-67.02.-38.18)
		(-85.97,-25.35)
		(-105.29,-19.38)
Italy	-39.101	(-51.92,-26.28)
		(-67.63,-16.36)
		(-90.01,-13.70)
Japan	-19.708	(-29.69,-9.72)
		(-42.01,-1.05)
		(-46.53,-4.33)
United Kingdom	-18.955	(-31.93,-5.98)
		(-40.19,-3.08)
		(-55.94,4.08)
G6	-44.204	(-55.60,-32.80)
		(-73.17,-23.62)
		(-82.87,-21.74)

Regression of $\hat{\Lambda}_t$ on $\hat{r}_t - \hat{r}_t^*$: $\hat{\Lambda}_t = \beta_0 + \beta_1(\hat{r}_t - \hat{r}_t^*) + u_{t+1}$

Country	$\hat{\beta}_1$	*90% c.i.(* $\hat{\beta}_1$*)*
Canada	23.610	(15.12,32.10)
		(12.62,51.96)
		(11.96,63.71)
France	13.387	(1.06,25.72)
		(-2.56,36.25)
		(-6.98,42.40)
Germany	34.722	(19.66,49.78)
		(9.34,57.59)
		(3.68,69.36)
Italy	27.528	(17.58,37.48)
		(14.98,48.32)
		(12.51,58.54)
Japan	15.210	(4.76,25.66)
		(-0.45,37.08)
		(0.91,38.87)
United Kingdom	14.093	(0.33,27.86)
		(0.39,34.46)
		(-8.70,46.45)
G6	31.876	(20.62,43.13)
		(16.89,54.62)
		(16.78,60.89)

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(Real interest rates themselves are estimates)

Difference between $-R_{t+k}$ and q_{t+k} is Λ_{t+k}:

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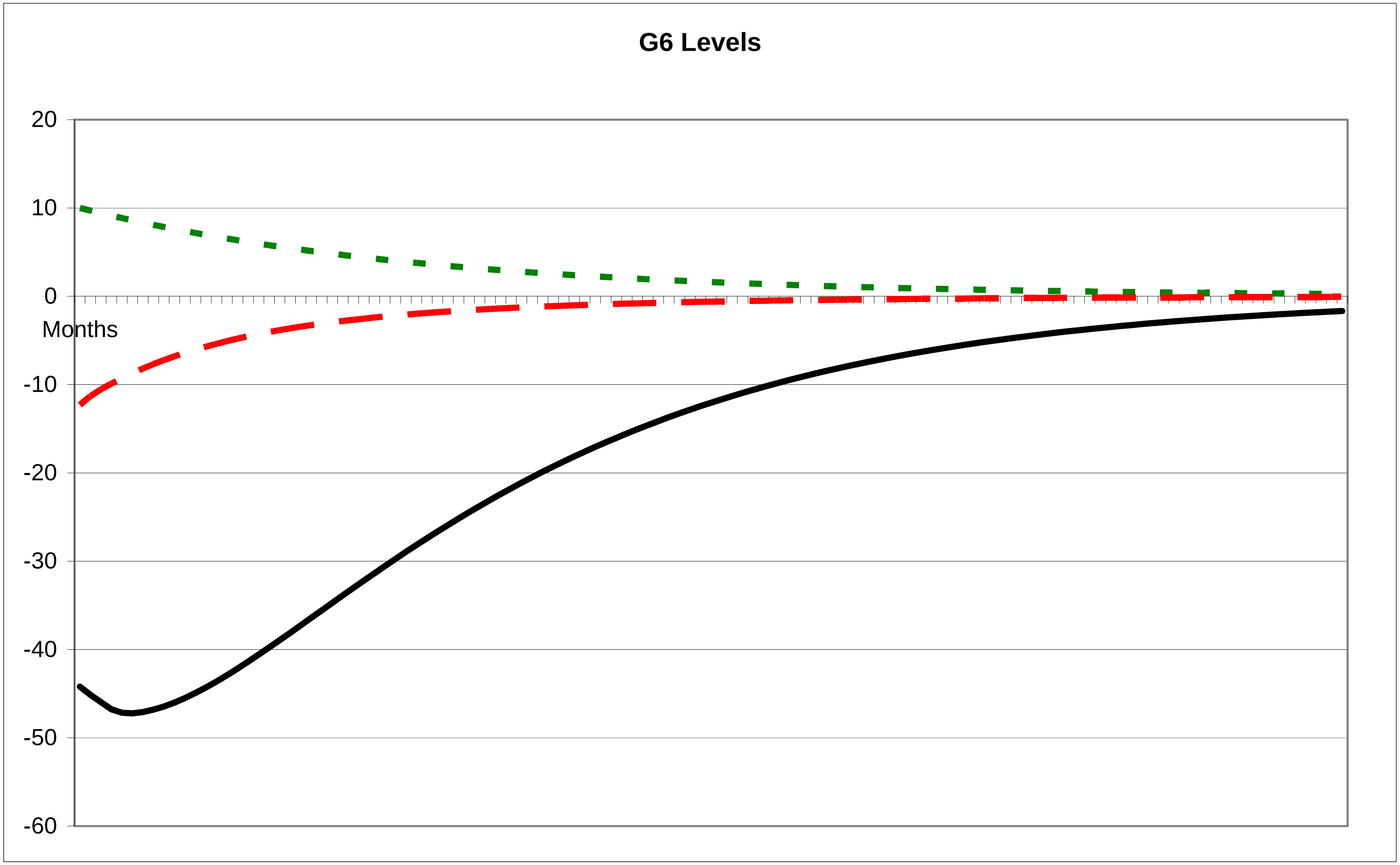


Implications:

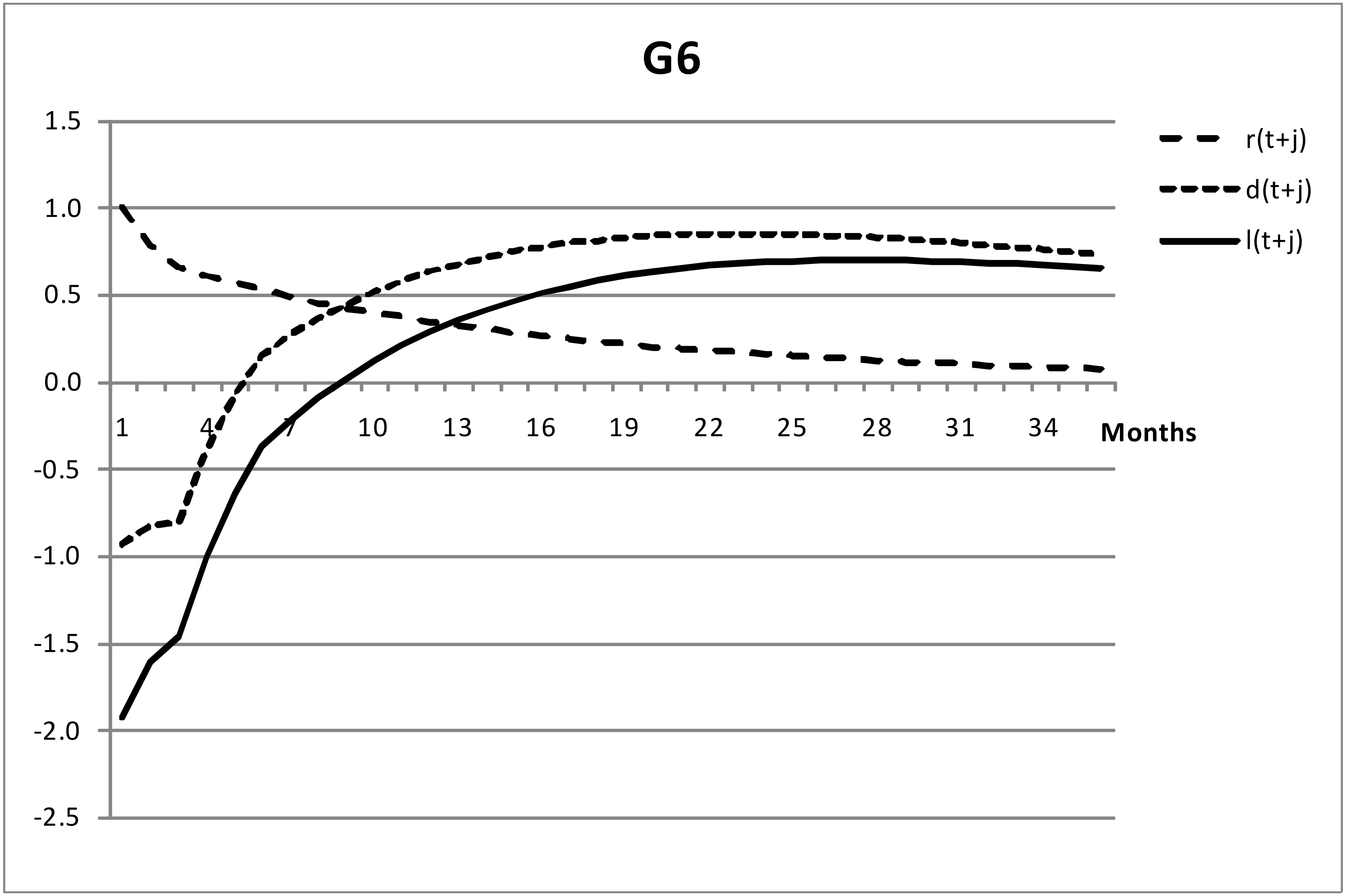
$\text{cov}(\lambda_t, r_t - r_t^*) < 0$ (Fama regression in real terms)

$\text{cov}(\Lambda_t, r_t - r_t^*) = \text{cov}(\sum_{j=0}^{\infty} E_t \lambda_{t+j}, r_t - r_t^*) > 0$ (from VAR estimates)

→ $\text{cov}(E_t \lambda_{t+j}, r_t - r_t^*) > 0$ for some *j* (as in previous figure)

Explaining $\text{cov}(\lambda_t, r_t - r_t^*) < 0$ and $\text{cov}(E_t \lambda_{t+j}, r_t - r_t^*) > 0$ is a challenge for risk premium models – when $r_t - r_t^*$ is high, the home currency is both riskier than average and expected to be less risky than average.

$$E_t\hat{d}_{t+k} = \beta_{dk}(\hat{r}_t - \hat{r}_t^*),\ E_t(\hat{r}_{t+k} - \hat{r}_{t+k}^*) = \beta_{rk}(\hat{r}_t - \hat{r}_t^*), \text{ and } E_t\hat{\lambda}_{t+k} = \beta_{\lambda k}(\hat{r}_t - \hat{r}_t^*)$$



In fact, models are built to account for $\beta_1 < 0$ in Fama regression and have $E_t \hat{q}_{t+1} - \hat{q}_t = \beta_1(\hat{r}_t - \hat{r}_t^*)$

imply $q_t - \lim_{j\to\infty} E_t q_{t+j} = -\beta_1 \sum_{j=0}^{\infty} E_t(\hat{r}_{t+j} - \hat{r}_{t+j}^*)$,

So that a higher home real interest rate depreciates the currency.

That goes against evidence, against the way we think about exchange rates.

But that is what gives level effect in the “model” line in Figure 1 the wrong sign.

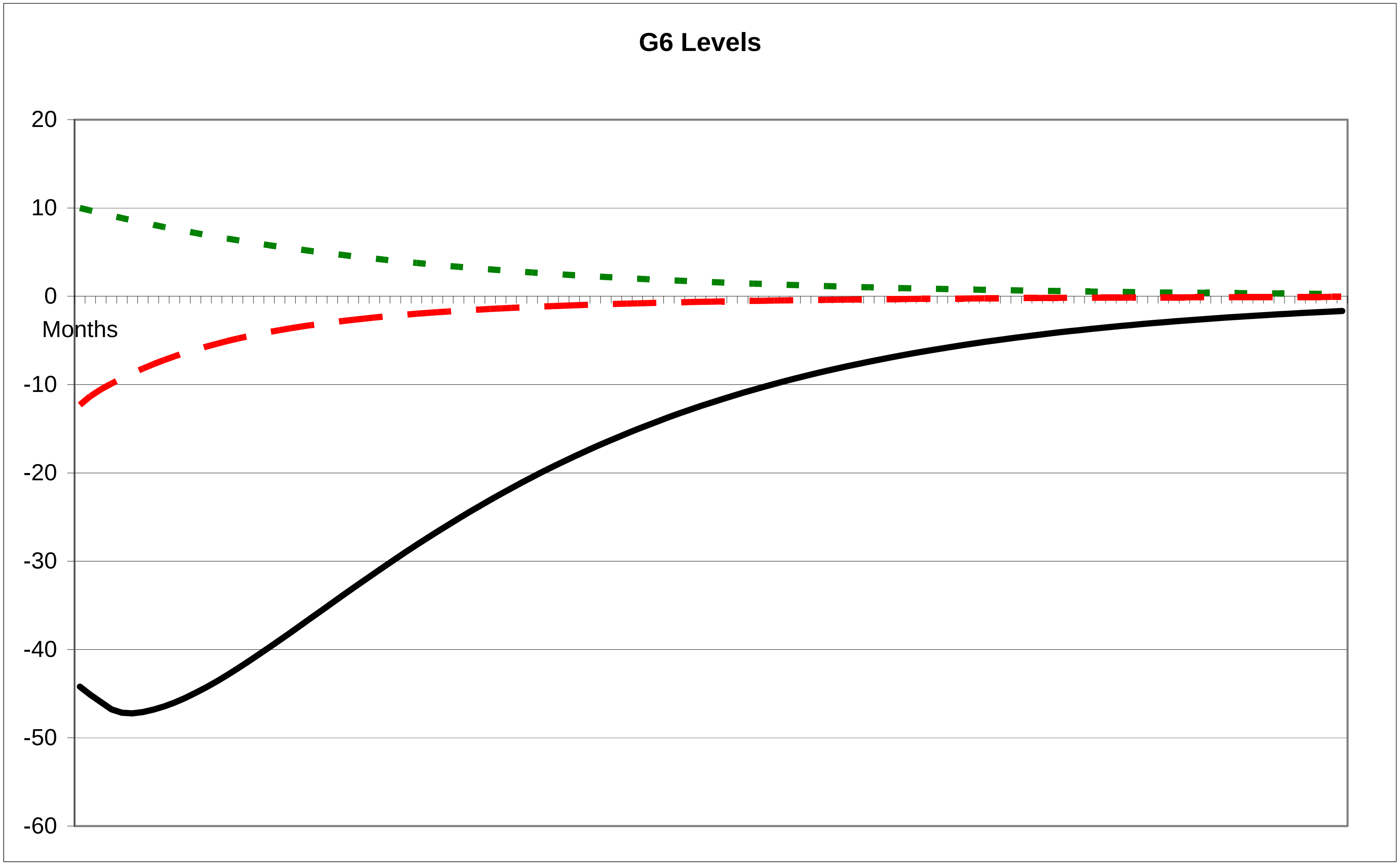
Models assume a single factor for risk premium and real interest differential:

$$E_t \hat{q}_{t+1} - \hat{q}_t = a_1 \phi_{1t}$$
$$\hat{\lambda}_t = g_1 \phi_{1t}$$

We need $\mathrm{var}(\lambda_t) > \mathrm{var}(E_t q_{t+1} - q_t)$ -- (Fama, 1984)

If we normalize $a_1 > 0$, we need $g_1 > a_1 > 0$.

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*),\ -R_{t+k} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*),\text{ and Model } q_{t+k} - \lim_{j\to\infty} E_t q_{t+j}$$



We need at least two factors:

$$E_t\hat{q}_{t+1} - \hat{q}_t = a_1\phi_{1t} + a_2\phi_{2t}$$
$$\hat{\lambda}_t = g_1\phi_{1t} + g_2\phi_{2t}$$

We need

$g_1 > a_1 > 0 \qquad 0 < g_2 < a_2$

(and also need the second factor to be more persistent in order to maintain $\text{var}(\lambda_t) > \text{var}(E_t q_{t+1} - q_t)$.)

Not a big deal if these were all just arbitrary.

The challenge is when we relate them to preferences and consumption.

Review of foreign exchange risk premium

m_{t+1}, m^*_{t+1} are logs of home, foreign stochastic discount factors

$$\rightarrow \quad \lambda_t = \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m^*_{t+1})$$

$$\rightarrow \quad E_t d_{t+1} = E_t(m^*_{t+1} - m_{t+1})$$

So we have to have

$$E_t d_{t+1} = E_t(m^*_{t+1} - m_{t+1}) = a_1\phi_{1t} + a_2\phi_{2t}$$
$$\lambda_t = \tfrac{1}{2}(\text{var}_t\, m_{t+1} - \text{var}_t\, m^*_{t+1}) = g_1\phi_{1t} + g_2\phi_{2t}$$

The challenge is that we have carefully designed models of preferences and the consumption process to give us $g_1 > a_1 > 0$. This gives us a ***volatile risk premium***, which we need for lots of asset pricing puzzles.

Now we also need $0 < g_2 < a_2$. Goes opposite direction…

Campbell-Cochrane preferences (Verdelhan, 2010):

$E_t q_{t+1} - q_t = \gamma(1-\phi)\sigma_t$

σ_t is deviation of consumption from external habit level

$\gamma > 0$ is risk aversion, $0 < \phi < 1$ is persistence of σ_t

$\lambda_t = [\gamma(1-\phi) - B]s_t$, where B is a parameter assumed to be < 0.

Hence, λ_t responds more than $E_t q_{t+1} - q_t$ to changes in s_t.

"Long-run Risks" (Bansal-Yaron (2004), Bansal-Shaliastovich (2010))

$-m_{t+1} = (\eta/2)w_t + \phi\sqrt{w_t}\,\varepsilon_{t+1}$ (+ other stuff)

w_t is the variance of persistent component of consumption growth.

The model assumes Epstein-Zin preferences. η and ϕ are determined by the parameters of risk aversion and intertemporal elasticity of substitution.

Under Epstein-Zin preferences, a preference for early resolution of risk ends up giving us $\eta < \phi^2$

$$E_t q_{t+1} - q_t = E_t(m^*_{t+1} - m_{t+1}) = (\eta/2)(w_t - w^*_t)$$
$$\lambda_t = \tfrac{1}{2}(\mathrm{var}_t\, m_{t+1} - \mathrm{var}_t\, m^*_{t+1}) = (\phi^2/2)(w_t - w^*_t)$$

Conclusion on risk premium models:

They will not explain the evidence of this paper.

They have been "reverse engineered" to deliver response of $\mathrm{var}_t\, m_{t+1}$ to state variables is greater than response of $E_t m_{t+1}$.

This feature is needed not only for UIP puzzle but for other asset pricing puzzles in equity and bond markets. These puzzles require large and variable risk premiums.

Delayed overshooting

Models with at least two types of agents (BvW, AAK)

One type is slow to react to shocks

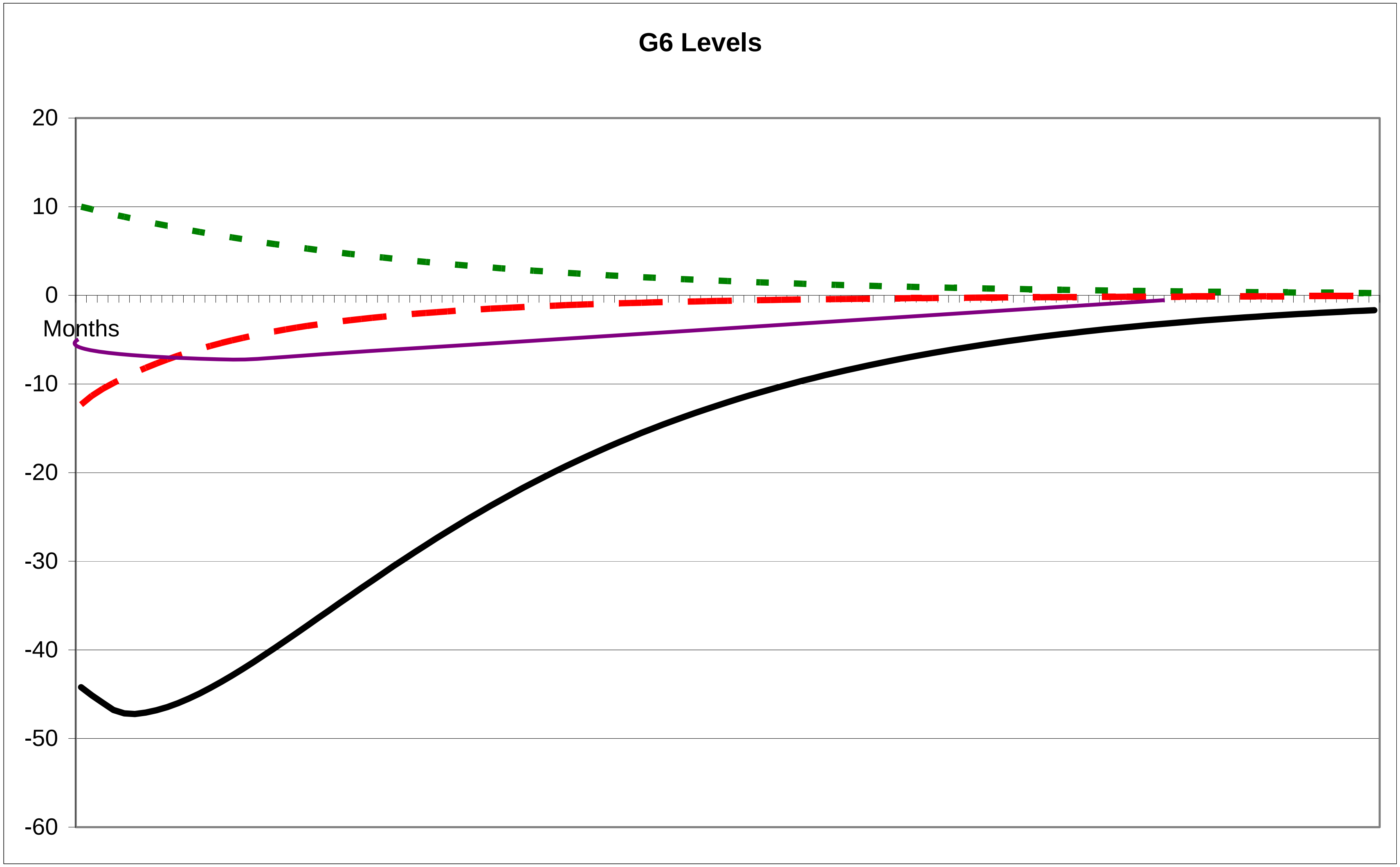
The other type moves quickly but is risk averse

When $r_t - r_t^*$ is high, q_t falls as in UIP.
It keeps falling in period $t+1$ because the slow-reacting agents move.

This leads to initial underreaction of q_t to increase in $r_t - r_t^*$.

Some tweaking may work – but markets are not efficient here.

$$q_{t+k} = \alpha_{qk} + \beta_{qk}(r_t - r_t^*),\ -R_{t+k} = \alpha_{Rk} + \beta_{Rk}(r_t - r_t^*),\ \text{and Model } q_{t+k} - \lim_{j\to\infty} E_t q_{t+j}$$



Conclusions:

A new puzzle. Our models for the UIP puzzle don't seem adequate.

This matters. Policy considerations:

1. We need to know how q_t reacts to changes in $r_t - r_t^*$ to assess monetary policy.

2. If indeed there is "overreaction", "momentum trading", or "delayed overshooting" in foreign exchange markets, this inefficiency translates into resource misallocation through its effects on relative goods prices.