Essentiality of money and money demand in explosive hyperinflation

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Abstract: This paper shows firstly that the money-in-the-utility-function model presents exactly the same kind of limitations as the cash-in-advance model for characterizing explosive hyperinflation. These limitations relate to sufficient money essentiality in the sense of Scheinkman (1980). Thereby, this paper contributes to the understanding of the Cagan inflationary finance models failure with perfect foresight. Secondly, the paper provides theoretical support to alternative functional forms of money demand that may be give an alternative to the failure of Cagan based inflationary finance model for the analysis of explosive hyperinflation. Theoretical support is brought to inelastic functional forms of money demand and specifically to the double-log schedule.

Keywords: monetary hyperinflation, inflation tax, money essentiality

JEL Code: E31, E41

1. Introduction

The main stylized facts provided by classical studies of hyperinflationary episodes (Bresciani-Turroni, 1937; Cagan, 1956; Sargent, 1982) characterize hyperinflation as a speeding up inflation unstable dynamic process where real money balances tend to vanish and the public deficit is financed by issuing money: these processes can be qualified as monetary hyperinflations.[1] Extreme inflation dramatically change economic exchange patterns compared to low-inflation inflation periods. The rapid depreciation of money during hyperinflation induces agents to spend money as soon as they have got it (Casella and Feinstein, 1990). Hyperinflation induces instability of relative price movements leading to large uncertainty about the outcomes of long-term contracts (Tang and Wang, 1993). As a consequence, hyperinflation decreases credit transactions and in general the use of long term contracts. This implies that money becomes more essential for purchasing goods during hyperinflation than during stable periods.

Scheinkman (1980) provided a precise definition of money essentiality. According to the latter, money is considered as essential if the inflation tax collected by the government does not tend to zero when the rate of inflation explodes. Therefore, the hyperinflation process is closely related to money essentiality and the inflation tax. Consistently with its salient stylized facts traditional models of hyperinflation view hyperinflation as the result of an inflationary finance policy. These inflationary finance models, such as Evans and Yarrow (1981) or Bruno and Fischer (1990), relying on the famous Cagan (1956) money demand, consider hyperinflation as a speeding up inflation process driven by an accelerating rise in the money supply as a means of raising revenues for the government by using the inflation tax. However, since the ‘surprising monetarist arithmetic’ analysed in Buiter (1987) it is known that under perfect foresight these models are fundamentally flawed because they are not capable of generating accelerating inflation.[2] Gutierrez and Vazquez (2004), resorting to first principles, consider two standard optimizing monetary setups modelling the transaction role of money: a money-in-the-utility-function model (henceforth called MIUF model) and a cash-in-advance model (henceforth called CIA model). They show that hyperinflationary dynamics derived from these standard optimizing monetary models may be consistent with a characterization of hyperinflation as an explosive process and perfect foresights.

This paper considers general setups of optimizing monetary MIUF and CIA models, and uses the precise definition of money essentiality given by Scheinkman (1980) with the aim to establish a formal theoretical link between the possibility of hyperinflationary paths and the concept of money essentiality. Modelling monetary hyperinflation with perfect foresight is shown to be closely linked to the concept of money essentiality in the formal sense of Scheinkman (1980). The main contribution of this paper is to show that, whether in a CIA or in a MIUF framework, this sufficient level of money essentiality is always conveyed by the representative agent’s preferences represented by its utility function and does not depend on the specific way, CIA or MIUF, of modelling the role of money as a medium of exchange. In this respect the paper contributes to the understanding of the well known

[1] This paper is not about speculative hyperinflations which are the focus of other works such as Brock (1975), Obstfeld and Rogoff (1983), Barbosa and da Cunha (2003) for instance. Speculative hyperinflations, as defined by Obstfeld and Rogoff (1983), are explosive price-level paths unrelated to monetary growth.

[2] Evans (1995) and Vazquez (1998) provide a survey about the literature concerning this failure.

Cagan inflationary finance models failure with perfect foresight by providing guidance about the choice of functional form of money demand for the analyse of explosive hyperinflation.

The paper is organized in the following way. Section 2 considers the general version of a MIUF economy and provides a general characterization of agents' preferences compatible with explosive hyperinflation relying on the concept of money essentiality. Section 3 studies the general version of a CIA model and shows again the dependence of explosive hyperinflation paths on a sufficient level of money essentiality. Section 4 relates money essentiality to money demand inelasticity and provides specific theoretical support to the double-log functional form of the money demand during hyperinflation. Section 5 concludes.

2. MIUF economy, hyperinflation and money essentiality

Both sections 2 and 3 adopt the basic setup of Gutierrez and Vazquez (2004) but extend it, first, by considering general utility functions and, second, by taking into account the goods market equilibrium condition.

The optimizing monetary models assume a continuous time model where the economy consists of a large number of identical infinitely-lived forward looking households endowed with perfect foresight. Population is constant and its size is normalized to unity for convenience. There is no uncertainty. Each household has a non-produced endowment $y_t > 0$ of the non-storable consumption good per unit of time.

In the MIUF model the role of money as a medium of exchange is assumed to be captured by introducing real money balances into the household utility function. Our framework considers agents preferences represented by a general class of utility function. Therefore, the representative household utility at time 0 is

$$\int_0^{\infty} U\left(c_t, m_t\right) e^{-rt} dt \,. \tag{1}$$

The instantaneous utility function has standard properties: it is continuous, twice differentiable on $\mathbb{R}_+^2$, increasing and strictly concave in c_t, the household's consumption at time *t*, and $m_t = \frac{M_t}{P_t}$ his holdings of real monetary balances (*M* is the nominal stock of money, *P* is the price level). The rate r is the subjective discount rate which is assumed to be equal to the real rate of interest. Financial wealth and the nominal interest rate are defined as

$$w_t = m_t + b_t \,,$$

$$i_t = r + \pi_t \,,$$

respectively, where b_t denotes real per capita government debt, π_t is the inflation rate. The household's budget constraint is

$$\dot{w} = y_t - \tau_t + rw_t - \left(c_t + i_t m_t\right), \tag{2}$$

where τ_t is a lump-sum tax assumed to be constant. The household's optimization problem leads to the following first-order condition:

$$r + \pi_t = \frac{U'_m(c_t, m_t)}{U'_c(c_t, m_t)}, \tag{3}$$

where $U'_c(c_t, m_t)$ is constant with respect to time because the instantaneous rate of time preference is equal to the real rate of interest. Condition (3) requires that at each moment the nominal rate of interest be equal to the marginal rate of substitution of consumption for money. It implicitly defines a demand for money as a function of the nominal interest rate *i*. The optimum solution is completed by the transversality condition:

$$\lim_{t \to \infty} \left[e^{-rt} U'_c(c_t, m_t) w_t \right] = 0 . \tag{4}$$

The setup is completed by considering the equilibrium condition in the goods market. Following Barbosa et al (2006) or Vazquez (1998, p. 438) “in the spirit of the traditional approach to the study of hyperinflationary phenomena, we assume that output and government expenditures are constant”. Therefore, the market for goods is in equilibrium when constant supply of good *y* equals household consumption and constant government expenditures (*g*):

$$y = c_t + g . \tag{5}$$

In usual inflationary finance models a constant per capita share of government’s budget deficit, *d*, is financed by issuing high-powered money:

$$d = \frac{\dot{M}_t}{P_t} = \dot{m}_t + \pi_t m_t . \tag{6}$$

Substituting the value of π extracted from first-order equation (3) in the latter expression leads to the inflationary finance model dynamics described by the following law of motion for real cash balances:

$$\dot{m}_t = d - \left(\frac{U'_m(c_t, m_t)}{U'_c(c_t, m_t)} - r \right) m_t . \tag{7}$$

Differential equation (7) provides a complete characterization of real per-capita money balances dynamics which will be studied by using the technique of phase diagram on $\left[0; +\infty\right[$. The main interesting point here is to examine whether this law of motion for real cash balances is able to produce hyperinflation paths. An explosive hyperinflation path will be observed if the law of motion presents a path leading to a zero level of real cash balances. Therefore, the conditions for this kind of paths should be identified. As the mathematical function representing the law of motion is continuous (which is true with standard assumptions on *U*) this kind of paths will be observed as long as (dropping index time for convenience):

$$\lim_{m \to 0_+} \dot{m} < 0 . \tag{8}$$

The calculation of $\lim_{m \to +\infty} \dot{m}$ will assess the existence of any steady state. Nevertheless, whatever the number of steady states, since we focus on possible explosive hyperinflation paths we are only interested in the paths starting at the left of the first possible steady state when the condition $\lim_{m \to 0_+} \dot{m} < 0$ is met.

At this stage a second highly important point should be made clear. According to Obstfeld and Rogoff (1983) in the context of speculative hyperinflations issue, any path leading to a zero value of real cash balances and crossing eventually the vertical axis at some finite point should be ruled out on grounds that such paths would not be feasible because the real stock of money would eventually become negative. However, we would rather follow the point made by Barbosa and Cunha (2003, p. 192) who contested the Obstfeld and Rogoff (1983) approach by arguing that on such hyperinflationary paths “when the real quantity of money reaches zero hyperinflation would have wiped out the value of money, the opportunity cost of holding money would have become infinite”, and “the economy would no longer be a monetary economy”. Therefore, we follow the point made by Barbosa and Cunha (2003) and consider the explosive hyperinflation paths corresponding to the condition $\lim_{m\to 0} \dot{m} < 0$ as perfect foresight competitive equilibrium paths.

Moreover, it’s important to stress that the possible explosive hyperinflationary paths are explosive monetary hyperinflations because along these paths the rate of growth of the money supply explodes. Rewriting government budget constraint as

$$\frac{\dot{M}}{M} = \frac{d}{m},$$

we see that along the paths of continuously declining m, given that $d > 0$, the growth rate of money supply increases continuously.

In this respect, according to the law of motion (7), the possibility of explosive hyperinflation will depend on the condition

$$\lim_{m\to 0}\left[\frac{U'_m(c,m)}{U'_c(c,m)}m\right] > d\ . \tag{9}$$

The latter condition is basically a condition about a sufficient level of money essentiality. In the sense of Scheinkman (1980) money is considered as essential if the inflation tax collected by the government does not tend to zero when the rate of inflation explodes. The interpretation is that “no matter how expensive it becomes to hold money people still hold a large quantity of it; that is money is very necessary to the system” (Scheinkman, 1980, p. 96). From (6) we see that seigniorage obtained by printing money can be decomposed into two components, the change in the real stock of money and the inflation tax πm which can be written, according to equation (3):

$$\pi m = \left(\frac{U'_m(c,m)}{U'_c(c,m)} - r\right)m = \frac{U'_m(c,m)}{U'_c(c,m)}m - rm\ .$$

Then, when the rate of inflation explodes we consider

$$\lim_{m\to 0}\pi m = \lim_{m\to 0}\left[\frac{U'_m(c,m)}{U'_c(c,m)}m\right].$$

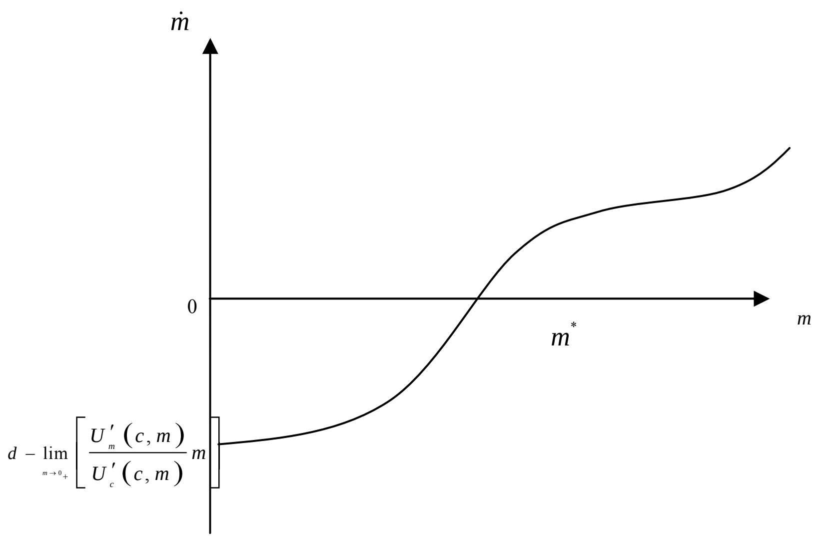
Therefore, when $\lim_{m\to 0}\left[\frac{U'_m(c,m)}{U'_c(c,m)}m\right] > 0$ then $\lim_{m\to 0}\pi m > 0$ and money is essential. These findings enable to formulate a first proposition.

Proposition 1: *In a general MIUF economy, explosive monetary hyperinflations are possible only if money is sufficiently essential that is if* $\lim_{m \to 0_+} \left[\frac{U'_m(c,m)}{U'_c(c,m)} m \right] > d$.

Proof: The proof relies on the previous arguments and can be illustrated by the phase diagram depicted on Figure 1. The precise shape of the phase diagram depends on the first and second derivative of $\dot{m}$ with respect to *m*. Other shapes than that depicted on Figure 1 could be possible for the phase locus. However, as the important point for the analysis conducted here insists on the condition for $\lim_{m \to 0_+} \dot{m} < 0$, our analysis focuses only on the paths leading to a zero value of real cash balances. If $\lim_{m \to +\infty} \dot{m} > 0$, the locus $\dot{m}$ will cross the horizontal axis at least once. We consider here a unique unstable steady state m^* but the qualitative analysis for explosive hyperinflationary paths doesn't change in the case of more steady states. All paths originating at the right of m^* are hyperdeflationary paths that can be ruled out because violating the transversality condition (4). All paths starting to the left of m^* are explosive hyperinflations paths.■

Chart 1

Monetary dynamics in a MIUF economy with money sufficiently essential



Explosive hyperinflation paths starting at the left of m^* are equilibrium paths since they are consistent with equilibrium condition on the goods market (5). Along these paths of declining real cash balances real per capita consumption will remain constant at $c_t = y - g$ but households will suffer from an increasing loss of welfare representing the harmful effect of hyperinflation on the economy.

Considering the case where the utility function is additively separable in consumption and real cash balances:

$$U(c_t, m_t) = u(c_t) + v(m_t) .$$

The functions *u* and *v* are increasing in their arguments and strictly concave, the condition (9) of Proposition 1 resumes to

$$\lim_{m \to 0} \left[mv'(m) \right] > du'(c) \ . \tag{10}$$

In the latter condition the value of $u'(c)$ is constant with respect to time. Scheinkman (1980) related the condition $\lim_{m \to 0} mv'(m) > 0$ to the essentiality of money. The condition (10), as a particular case of Proposition 1, states that the possibility of explosive hyperinflation depends on a sufficient level of money essentiality which is conveyed by the utility function for money services.

According to Proposition 1, the failure of the Cagan inflationary finance model to produce explosive hyperinflations is not surprising. The Cagan ad-hoc model relying on the Cagan money demand can be considered as a special case of the MIUF model developed here. Since Kingston (1982), it is known that the semi-log schedule is 'integrable'. In the terms of the latter it means that the schedule 'can be generated by at least one optimizing framework'. The 'integrability' of Cagan money demand was shown again later by Calvo and Leiderman (1992).

Proposition 2: *Cagan money demand does not comply with money essentiality.*

Proof: The 'integrability' of Cagan money demand is shown by using a utility function for money services *v(m)* such as:

$$v\left(m\right) = \alpha^{-1}\left(1 + \gamma + \alpha r - \log m\right) m \quad \text{for all } 0 < m < e^{\gamma + \alpha r} \ .$$

The latter utility function for money services will deliver through the first-order equation (5) the famous semi-logarithmic Cagan money demand ($\log m = \gamma - \alpha\pi$, where γ is a constant and α a positive constant). The current MIUF model will resume in the inflationary finance Cagan model. However, such a utility function for money services doesn't comply with money essentiality requirement since for the latter utility function $\lim_{m \to 0} mv'(m) = 0$. Then, it won't allow the modelling of monetary hyperinflation as stated in Proposition 1.

3. CIA economy, hyperinflation and money essentiality

The CIA model considered here differs from that of the previous MIUF in two aspects. First, the representative household's preferences are represented by utility function depending only on the level of real consumption. Then, the household utility at time 0 is

$$\int_0^\infty e^{-rt} U(c_t) dt \ . \tag{11}$$

The function *U* belongs to a general class of utility function. It is increasing and strictly concave in its single argument, real good consumption. Second, in a CIA economy the role of money as a medium of exchange is captured by a CIA constraint assuming that money holding is strictly essential to buy the consumption good. In order to consume *c* units of the consumption good at time *t*, the household must hold a stock of real cash balances, *m*, greater or equal to *c*:

$$m_t \geq c_t \ .$$

Assuming the existence of an interior solution for *c*, and that the nominal interest rate *i* is greater than zero, meaning that money is return-dominated by government bond, it follows that CIA constraint must hold with equality:

$$m_t = c_t \ . \tag{12}$$

The representative household optimization problem consisting of maximizing (11) subject to the constraints given by (2) and (12) leads to the following first order condition:

$$U'(m_t) = \lambda\left(1+i_t\right). \tag{13}$$

The associated Lagrange multiplier λ is constant with respect to time because the agent's rate of time preference equals the real rate of interest, and real cash balances will indirectly enter the utility function according to (12). Equation (13) characterizes a demand for real money balances decreasing with respect to the rate of inflation (or the cost of holding cash balances) because the utility function U is strictly concave. The transversality condition implies that

$$\lim_{t\to\infty} e^{-rt}\lambda w_t = 0. \tag{14}$$

By using the definition of the nominal interest rate, the first order condition (13) can be written as follows:

$$\pi_t = \frac{U'(m_t) - \lambda\left(1+r\right)}{\lambda}. \tag{15}$$

As in usual inflationary finance models a constant per capita share of government's budget deficit, *d*, is financed by issuing high-powered money, the law of motion for real money balances in this CIA inflationary finance model will be given by combining (6) and (15), leading to

$$\dot{m}_t = d - \frac{1}{\lambda}\left(U'(m_t) - \lambda\left(1+r\right)\right)m_t, \tag{16}$$

On the basis of the methodology and the argumentation developed in section 2, the possibility of explosive hyperinflation paths depends on condition (8) leading to the following condition for the CIA economy (dropping the time index for convenience)

$$\lim_{m\to 0_+}\left[mU'(m)\right] > \lambda d. \tag{17}$$

In the same way as in section 2 in the framework of a MIUF economy, condition (17) relates the possibility of explosive hyperinflation to a sufficient level of money essentiality in the sense of Scheinkman (1980). Moreover, this sufficient level of money essentiality is conveyed by the agent's preferences. According to (15), inflation tax is given by

$$\pi m = \left(\frac{U'(m) - \lambda\left(1+r\right)}{\lambda}\right)m.$$

Then, when the rate of inflation explodes we consider

$$\lim_{m\to 0_+}\left[\pi m\right] = \frac{1}{\lambda}\lim_{m\to 0_+}\left[mU'(m)\right].$$

From the mathematical point of view it appears that condition (17) allowing the model to generate possible explosive hyperinflations paths is exactly of the same kind as condition (9) in the general MIUF model. Condition (17) is particularly similar to condition (10) in the MIUF case with an additive separable utility function.

Proposition 3: *In a CIA economy with a general class of utility function, explosive hyperinflations are possible only if money is sufficiently essential that is if* $\lim_{m\to 0_+}\left[mU'(m)\right] > \lambda d$.

Proof: The proof relies on previous arguments.■

The possibility of monetary hyperinflation paths is again a discussion about a sufficient level of money essentiality. It is not linked to the specificity of the CIA framework. The CIA framework is not sufficient in itself to ensure the possibility of explosive hyperinflations paths.

Explosive hyperinflation paths in the CIA economy raise an important issue. According to the CIA constraint (12), household real consumption will fall along explosive hyperinflation paths characterized by the declining value of real money balances. The fall of households' real consumption will cause an increasing loss of welfare and represent the harmful effect of hyperinflation on the CIA economy. There is some evidence supporting this result. As pointed out by Vazquez (1998), Webb (1989) in his Table 5.4 shows evidence that consumption fell dramatically during German hyperinflation. For instance, the consumption of butter, meat and sugar fell to 5%, 39% and 3% of the levels of consumption in 1913, respectively. Moreover, Bresciani-Turroni (1937, p. 329) describes how certain classes were hit by poverty during German hyperinflation.

Nevertheless, the goods market equilibrium condition (5) questions the validity of explosive hyperinflation paths as equilibrium paths in the CIA economy. According to equilibrium condition (5) household real consumption c should be constant at the level $c = y - g$ because endowment in the non storable good is constant at level y and government expenditures are constant at level g. Then, any explosive hyperinflation path in the CIA economy doesn't comply with goods market equilibrium condition and may not be considered as equilibrium path. It may be therefore ruled out as not being consistent with goods market equilibrium condition. That would seriously affect results obtained in Gutierrez and Vazquez (2004) and our Proposition 3.

Two solutions may be proposed to ensure the validity of explosive hyperinflation paths in the CIA economy. First, we could imagine that transactions not taking place in the monetary economy because of the declining value of real cash balances may be offset by an increasing resort to unofficial barter in the grey economy. Total real consumption at time t could then be split into two components $c_t = c_{1t} + c_{2t}$ where c_{1t} would represent consumption constrained by holding of real cash balances, and c_{2t} would represent real consumption achieved through unofficial barter in the grey economy. In that respect, equilibrium condition on goods market could be written as $y = c_{1t} + c_{2t} + g$. Then, along an explosive hyperinflation path c_{1t} would decrease with the real value of money balances and c_{2t} would increase consistently with goods market equilibrium meaning that more and more transactions would be performed in the grey zone. Eventually, the CIA monetary economy would collapse and switch entirely to an unofficial barter grey economy. Second, we could imagine that the fall of real consumption may generate the fall of goods supply leading eventually to the collapse of the economy. Then, at each time t goods supply y_t would adjust to the falling household real consumption c_t such that $y_t = c_t + g$. Current evidence provided by the collapsing Zimbabwean economy may support this possibility (see Table 1). Zimbabwean real GDP registered a drop of more than 40% since 1999.

Table 1

Economic crisis and inflation during Zimbabwean hyperinflation

Year	2002	2003	2004	2005	2006	2007	2008
GDP growth at constant prices (annual percent change)	-4,370	-10,363	-3,557 e	-3,953 e	-5,422 e	-6,092 e	-6,627 e
Inflation, average consumer prices (annual percent change)	133,215	365,046	349,988	237,817	1 016,683	10 452,55 e	n/a

Sources : IMF, e: IMF estimates.

A similar idea would be to consider that the fall of household consumption would lead the household endowment to be simply wasted and the economy to collapse eventually. We could view the model as one in which labour endowment had to be sold for fiat money and explosive hyperinflation would lead to the wasting of the households' labour endowment leading eventually to the collapse of monetary economy.

4. Money essentiality, money demand inelasticity and monetary hyperinflation

Money essentiality is closely related to the inelasticity of the demand for money with respect to the cost of holding cash balances. We define the function $f(m)$ measuring the cost of money services according to

$$f(m) = mi = m\left(r+\pi\right) = \begin{cases} m \cdot \dfrac{U'_m\left(c,m\right)}{U'_c\left(c,m\right)} & \text{in the MIUF economy} \\ m \cdot \left(\dfrac{U'\left(m\right)-\lambda}{\lambda}\right) & \text{in the CIA economy} \end{cases}.$$

The first derivative of $f(m)$ is

$$f'(m) = i\left(1+\frac{m}{i}\frac{\partial i}{\partial m}\right) = i\left(1-\frac{1}{|\varepsilon|}\right).$$

where ε represents the elasticity of the money demand with respect to the nominal interest rate. If the money demand is interest-rate inelastic, $|\varepsilon| < 1$, then $f'(m) < 0$.

Since $f(m) \geq 0$ and $f'(m) < 0$ when the money demand is inelastic, it follows that

$$\lim_{m\to 0} f(m) = \begin{cases} \lim\limits_{m\to 0} m \cdot \dfrac{U'_m\left(c,m\right)}{U'_c\left(c,m\right)} > 0 \text{ in the MIUF economy} \\ \dfrac{1}{\lambda}\lim\limits_{m\to 0} mU'\left(m\right) > 0 \text{ in the CIA economy} \end{cases}.$$

Therefore, when money demand is interest rate-inelastic, money is essential.

Proposition 4: *Any money demand function inelastic with respect to the cost of holding cash balances and such that* $\lim_{m \to 0} f(m) > d$ *will allow the modelling of monetary hyperinflation under perfect foresight.*

Proof: The proof relies on Proposition 1 and Proposition 3.■

Barbosa et al (2006), in a similar framework, point out the role of the inelasticity of money demand functions with respect to the nominal interest rate for the possibility of explosive inflation path but insist in the need of an increasing government deficit. Our results stress, rather, the role of money essentiality and are established with a constant government deficit without needing an increasing deficit.

Proposition 4 establishes that inelastic money demand function complying with a sufficient level of money essentiality can be candidates for replacing the famous Cagan money demand function to model successfully monetary hyperinflation under perfect foresight. Among them we may consider the double-log schedule:

$$\log m = \delta - \beta \log \pi, \qquad 0 < \beta < 1,$$

with δ constant. This money demand functional form exhibits a constant elasticity lower than one with respect to the inflation rate.

Proposition 5: *The double-log schedule is an appropriate candidate functional form to replace Cagan money demand function in the analysis of monetary hyperinflation under perfect foresight.*

Proof: As shown by Kingston (1982), the double-log schedule is 'integrable' in a MIUF setup. Using the setup of a MIUF economy with additive-separable utility function for instance, one can easily verify that using a utility function for money services *v(m)* such as

$$v(m) = \left(rm + \frac{\beta e^{\frac{\delta}{\beta}}}{\beta - 1} m^{1-\frac{1}{\beta}} \right) \cdot u'(c),$$

will give microeconomic foundations to the double-log schedule. The money demand function described by the double-log schedule complies with Proposition 1 as shown by the following calculation:

$$\lim_{m \to 0} \frac{mv'(m)}{u'(c)} = +\infty > d .$$

The double-log schedule is 'integrable' in the CIA setup of section 3 as well. Using a utility function such as

$$U(c) = \lambda(1+r)c + \lambda \frac{\beta}{\beta - 1} c^{1-\frac{1}{\beta}},$$

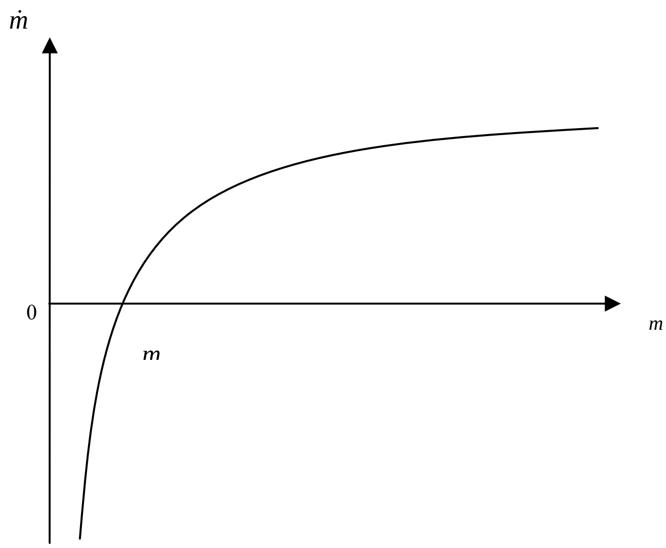
will also microeconomic foundations to the double-log schedule complying with Proposition 3 since:

$$\lim_{m \to 0_+} mU'(m) = +\infty > \lambda d \ .$$

Figure 2 represents the monetary dynamics derived from the double-log schedule under perfect foresight. All paths starting at the left of the unique unstable steady state m^* are monetary hyperinflations. The paths starting at the right of the unique steady state can be ruled out because violating the transversality condition (4).■

Chart 2

Monetary dynamics with the double-log schedule



Proposition 5 provides theoretical support for the use of the double-log schedule for money demand in the modelling of explosive hyperinflation under perfect foresight.

5. Conclusion

The first result of this paper is to show that the possibility of explosive hyperinflation paths in general setups of a MIUF and a CIA model depends on a sufficient level of money essentiality defined in the formal sense of Scheinkman (1980). In that respect we depart from Gutierrez and Vazquez (2004) by showing that the CIA model presents exactly the same kind of limitations as the MIUF model for characterizing explosive hyperinflation paths. CIA constraint doesn't convey by itself sufficient money essentiality even if it makes money necessary for the transactions. The sufficient level of money essentiality is conveyed by the representative agents' preferences[3].

[3] The sufficient money essentiality requirement is relevant for hyperinflationary paths analyse beyond technical arguments. As pointed out by Gutierrez and Vazquez (2004), money becomes more essential for purchasing goods during hyperinflation than during stable periods "because extreme inflation dramatically decreases credit transactions and in general the use of long term contracts".

Therefore, the paper may contribute to the understanding of the well known failure of Cagan inflationary finance models with perfect foresight. A particular class of inelastic money demand functions has been shown to be appropriate candidates to replace the Cagan money demand function in the analysis of explosive hyperinflation in inflationary finance models. More specifically, the paper provides theoretical support to the double-log schedule. Ashworth and Evans (1998) looking for alternative functional forms for money demand under hyperinflation provided empirical support for the double-log schedule as well. Therefore, the double-log schedule may be a possible and appropriate candidate functional form to give an alternative to the failure of Cagan based inflationary finance model for the analysis of explosive hyperinflation. Further research may be conducted for the choice of appropriate demands for real cash balances in hyperinflation contexts for which microeconomic foundations should comply with the money essentiality requirement.

Moreover, a sufficient level of money essentiality is crucial in inflationary finance models of hyperinflation since the government needs the money to be essential to the system in order to get sufficient inflation tax when inflation explodes.

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